Appendix

This appendix illustrates the derivation of velocity difference $v_{c,n} - v_{b,n}$ calculation in (11) in the paper.

Assuming that a small water droplet can be decomposed into $N_\omega + 1$ rigid infinitesimal sections, where ξ_c and ξ_b ($b = 1, \ldots, N_\omega$) are the infinitesimal sections including center point $\mathbf{w}_c$ and boundary point $\mathbf{w}_b$ respectively (cf. Fig. A-1a). Then, the velocities of center or boundary points $\mathbf{w}_c$, $\mathbf{w}_b$ can be represented by the velocities of infinitesimal sections ξ_c, ξ_b including them.

Let the center infinitesimal section ξ_c pass through a 3D tube with a constant time-axis velocity $v_\mathcal{C}$ (i.e., $v_{c,n} = v_\mathcal{C}, \forall n$), we need to calculate the time-axis velocity $v_{b,n}$ of boundary infinitesimal sections ξ_b.

For simplicity, we neglect the gravity forces on ξ_b and the interacting forces among ξ_b. The time-axis velocity of ξ_b is then jointly decided by the viscosity force from the center infinitesimal section ξ_c and the friction force from a 3D tube's boundary, as in Fig. A-1b. Thus, according to Newtons second law of motion, the force-and-motion correlation for ξ_b can be constructed by:

$$m_b \cdot a_{b,n-1} = \mathcal{F}^o_{b,n-1} - \mathcal{F}^f_{b,n-1} \tag{A-1}$$

where $a_{b,n-1}$ is the acceleration of boundary infinitesimal section ξ_b when it passes through boundary position $\mathbf{P}_{b,n-1}$. m_b is the mass of ξ_b which is assumed to be the same for all boundary infinitesimal sections (i.e., $m_b = m_\mathcal{B}, \forall b$). $\mathcal{F}^o_{b,n-1}$ and $\mathcal{F}^f_{b,n-1}$ are the time-axis viscosity and friction forces applied on ξ_b at position $\mathbf{P}_{b,n-1}$, respectively.

The acceleration $a_{b,n-1}$ in (A-1) can be calculated by:

$$a_{b,n-1} = \frac{v_{b,n} - v_{b,n-1}}{\Delta t} \tag{A-2}$$

where $v_{b,n}$ is the time-axis velocity of ξ_b when passing through position $\mathbf{P}_{b,n}$ ($\mathbf{P}_{b,n}$ is the next position of $\mathbf{P}_{b,n-1}$ on ξ_b's route). Δt is the time used to change from $v_{b,n-1}$ to $v_{b,n}$, and it is set as a constant for different $v_{b,n-1}$.

Furthermore, according to fluid viscosity theories [1], [2], the viscosity force $\mathcal{F}^o_{b,n-1}$ in (A-1) can be approximated by:

$$\mathcal{F}^o_{b,n-1} \approx \mu_1 \mathcal{S} \frac{v_\mathcal{C} - v_{b,n-1}}{r^s_{b,n-1}} \tag{A-3}$$

where $v_\mathcal{C}$ is the constant velocity of the center infinitesimal section ξ_c. μ_1 is the viscosity coefficient. $\mathcal{S}$ is the contact area between infinitesimal sections ξ_c and ξ_b, and it is set as a constant for all ξ_b (cf. Fig. A-1b). $r^s_{b,n-1}$ is the distance between center and boundary positions $\mathbf{P}_{c,n-1}$ and $\mathbf{P}_{b,n-1}$ (cf. (11) in the paper). $r^s_{b,n-1}$ is utilized to approximate the average distance between points in infinitesimal sections ξ_b and ξ_c (cf. Fig. A-1b). According to (A-3), the viscosity force is controlled by the thickness of a 3D tube. When a water droplet passes through a thick tube, $r^s_{b,n-1}$ becomes large and the viscosity force from ξ_c to ξ_b becomes small.

Similarly, the time-axis friction force $\mathcal{F}^f_{b,n-1}$ can be approximated by:

$$\mathcal{F}^f_{b,n-1} \approx \mu_2 (1 + \cos\theta_{b,n-1}) \tag{A-4}$$

where μ_2 is the constant friction coefficient [3]. $\theta_{b,n-1}$ evaluates the relative location of a boundary position $\mathbf{P}_{b,n-1}$ with respect to the motion route of a water droplet (cf. (11) in the paper). $1+\cos\theta_{b,n-1}$ is utilized to model the effect of a water droplet's normal force on tube boundaries. For example, in Fig. A-1b, when a water droplet is moving leftward in horizontal direction, it is modeled to provide larger normal forces on tube boundary points $\mathbf{P}_{1,n-1}$ on the left. This leads to a larger friction force applied on infinitesimal section ξ_1. In this way, the velocities of boundary infinitesimal sections can be controlled by the route of a 3D tube. Consequently, the route information of a 3D tube can be effectively embedded in the output droplet shape (cf. Fig. 6 in the paper).

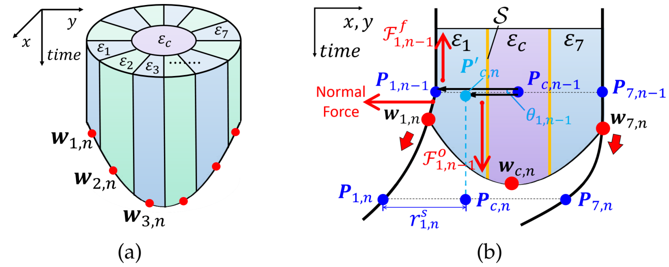

Figure A-1. (a) Illustration of infinitesimal sections ξ_c and ξ_b; (b) Illustration of viscosity and friction forces $\mathcal{F}^o_{b,n-1}$ and $\mathcal{F}^f_{b,n-1}$, contact area $\mathcal{S}$ between infinitesimal sections, distance $r^s_{b,n}$ between center and boundary positions, and relative angle $\theta_{b,n-1}$ of a boundary position to a droplet's motion route.

Based on (A-2) to (A-4), (A-1) can be rewritten by:

$$m_\mathcal{B} \frac{v_{b,n} - v_{b,n-1}}{\Delta t} = \mu_1 \mathcal{S} \frac{v_\mathcal{C} - v_{b,n-1}}{r^s_{b,n-1}} - \mu_2 - \mu_2 \cos\theta_{b,n-1} \tag{A-5}$$

After omitting the constant term μ_2 in (A-5)[1], we can easily derive the time-axis velocity $v_{b,n}$ of ξ_b when passing through position $\mathbf{P}_{c,n}$ by:

$$v_{b,n} \approx \left(1 - \frac{\lambda_1}{r^s_{b,n-1}}\right) v_{b,n-1} + \frac{\lambda_1}{r^s_{b,n-1}} v_\mathcal{C} - \lambda_2 \cos\theta_{b,n-1}$$

where $\lambda_1 = \frac{\mu_1 \mathcal{S} \Delta t}{m_\mathcal{B}}$ and $\lambda_2 = \frac{\mu_2 \Delta t}{m_\mathcal{B}}$ are constant coefficients.

Finally, the velocity difference $v_{c,n} - v_{b,n}$ in (11) in the paper can be calculated as:

$$v_{c,n} - v_{b,n} \approx \left(1 - \frac{\lambda_1}{r^s_{b,n-1}}\right) (v_\mathcal{C} - v_{b,n-1}) + \lambda_2 \cos\theta_{b,n-1} \tag{A-6}$$

References

[1] R. H. Pletcher, J. C. Tannehill, and D. Anderson, *Computational fluid mechanics and heat transfer*. CRC Press, 2012.

[2] S. Patankar, *Numerical Heat Transfer and Fluid Flow*. CRC Press, 1980.

[3] B. Armstrong-Helouvry, *Control of machines with friction*. Springer Science & Business Media, 2012.

1. Note that when plotting droplet figures in Figs 5, 10 in the paper, the constant term is not omitted in order to display d^t_b in positive values.