Joint Pricing and Decision-Making for Heterogeneous User Demand in Cognitive Radio Networks

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***Abstract*—The cognitive radio technique allows secondary users (SUs) to share the spectrum with primary users (PUs) in an exclusive or opportunistic manner. This paper studies spectrum pricing conducted by spectrum owners, that is, primary operators (POs), and SU decision-making strategies for three kinds of duopoly markets. The single-band exclusive use market considers two POs with each providing a single band dedicated to SUs. A pre-emptive resume priority (PRP) M/M/1 queueing model is presented, based on which SUs decide to join which PO and which queue. We prove the existence of a unique Wardrop equilibrium for the decision-making process, and a unique Nash equilibrium for the proposed parallel pricing strategy. In a single-band mixed use market, the competition of two POs is represented by a Stackelberg game. We formulate the spectrum sharing among PU and SUs with a 3-level PRP M/M/1 queueing structure, and derive the close form expressions of SUs' queueing delay. In a multiband exclusive use market, where POs have to determine how many bands they will rent as well as the admission price, we define the problem as a mixed integer linear programming problem and propose a global particle swarm optimization algorithm to find the global optimum. Finally, we study a generalized scenario with multiple POs and multiple priority queues.**

***Index Terms*—Cognitive radio (CR) networks, decision-making, dynamic spectrum access (DSA), Nash equilibrium (NE), pricing, Wardrop equilibrium (WE).**

I. Introduction

A REPORT from the Federal Communications Commission [1] reveals a fact that the emerging "spectrum shortage" comes from the inefficient spectrum usage rather than the real spectrum scarcity. Cognitive radio (CR) or dynamic spectrum access (DSA) [2], has been proposed to improve the spectrum efficiency. DSA allows secondary users (SUs) to access the spectrum licensed to primary users (PUs) in an opportunistic manner. This way, on one hand, improves the spectrum utilization by enabling spectrum sharing between primary and SUs. On the other hand, primary operators (POs) can earn the revenue by leasing their spectrum which is idle temporarily to SUs.

Under the current CR structure, SUs can use the spectrum by several DSA models, such as exclusive use, open sharing, and opportunistic access [3]. Among these models, the exclusive use and opportunistic access have been widely considered in [4]–[6]. The exclusive use model allows POs to lease parts of unused spectrum dedicatedly to SUs without service interruption from PUs. The opportunistic access requires SUs to perform spectrum sensing [7] and utilize spectrum holes that are not used by PUs in a nonintrusive manner.

In existing studies on DSA strategy design, SUs are often assumed to select the spectrum for maximizing the achievable transmission rate or bandwidth [8]–[15], in which two issues are generally not considered. One is that SUs carrying different services would have different quality of service (QoS) requirements. The other is that as some SUs may select the same PO band simultaneously, a queueing model is required to characterize the waiting time arising from channel contention of multiple SUs. In this paper, we consider SUs heterogenous sensitive to congestion, and study PO pricing and SU decision-making strategies based on queueing theory. In such case, the spectrum selection behavior of SUs can be modeled as a nonatomic game, whose stable outcome is a Wardrop equilibrium (WE) [16], at which no SU can improve the tradeoff between the average queueing delay and the monetary cost by unilaterally changing its decision.

In this paper, we study three kinds of duopoly markets: 1) single-band exclusive use; 2) single-band mixed use; and 3) multiband exclusive use. Unlike the monopoly and duopoly markets discussed in [17]–[21], we consider more practical and generalized scenarios, including the exclusive use and opportunistic access mixed spectrum sharing, multiple priority queues for heterogeneous SU demand, multiband spectrum sharing, and finally extend to a generic multi-PO market.

The single-band exclusive use market considers two POs with each providing a single band for exclusive use. To guarantee the fairness, a parallel pricing strategy is presented

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that allows two POs to set their prices simultaneously. Each PO band serves two priority queues for delay sensitive and insensitive SU demand. A pre-emptive resume priority (PRP) M/M/1 queueing model [22] is developed, based on which the SU decides to which queue as well as which PO band it joins. We prove that a unique WE is reached by the decision-making process. Also, we show the existence of a unique Nash equilibrium (NE) for the parallel pricing strategy.

In single-band mixed use market, one PO provides exclusive use and the other provides opportunistic access. Different from the exclusive use scenario, we describe the price competition between two POs as a two-stage Stackelberg game, in which the PO offering exclusive band use acts as the leader, while the PO providing opportunistic access plays as the follower. For opportunistic access model, we formulate the spectrum sharing among PU, SUs in high priority (HP) queue, and SUs in low priority (LP) queue with a 3-level PRP M/M/1 queueing structure, and derive the close form expressions for the expected queueing delay of SUs.

Finally, we discuss a more complicated case of multiband exclusive use, where each PO has multiple idle bands that can be exclusively used by SUs. In this case, besides setting the band price, the POs have to determine how many bands they will lease. We formulate the performance tradeoff between revenue maximization and cost minimization as a mixed integer linear programming (MILP) problem based on an M/M/s queueing model. In order to find a polynomial-time solution, we decompose the MILP problem into a two-level optimization procedure, including a pricing problem and an integer programming (IP) problem for finding the number of bands to be leased. Further, a global particle swarm optimization (GPSO) algorithm is developed to find the global optimum of the IP problem, in which a spectrum broker is required to execute the GPSO algorithm in a centralized way.

The remainder of this paper is organized as follows. Related work is reviewed in Section II. The network scenarios and queueing models are described in Section III. The pricing and decision-making strategies and their numerical results are presented in Sections IV–VI, respectively, for three duopoly markets. Section VII extends the discussion to a generalized scenario with multiple POs and multiple priority queues. Section VIII concludes this paper.

II. Related Work

Pricing-based spectrum access control in CR networks has been extensively studied in [8]–[15]. In most of these studies, SUs select the spectrum on the basis of the achievable transmission rate, bandwidth, energy efficiency, as well as the admission price. For instance, D'Oro *et al.* [8] studied the problem of cost-efficient throughput maximization in multicarrier CR networks, and analyzed SUs' reaction to PUs' charge for maximizing the achievable transmission rate. A differentiated pricing strategy is proposed in [12], which charges SUs in terms of their different spectrum demands, and SUs purchase the optimal amount of spectrum for maximum bandwidth. In [14], a linear pricing technique is presented for OFDMA CR networks, where secondary transmitters choose the number of subchannels and transmit power to maximize their own energy efficiency. In this paper, we take into account SUs' heterogenous sensitive to congestion, and focus on the competition for congestible resource by coupling game theory, queuing theory, and decision theory.

Recently, interactions between pricing and decision process based on queueing systems have attracted interests in CR networks [17]–[21]. For example, Tran *et al.* [18] modeled two kinds of monopoly price competitions, where the queueing delay of SUs are established in an M/G/1 queueing system including two priority queues. Elias *et al.* [20] formulated a duopoly competition between PO (offering a guaranteed service) and secondary operator (offering a best-effort service), and analyzed the service selection strategies of SUs on the basis of an M/M/1 queueing model. Do *et al.* [21] considered pricing strategies in both monopoly and duopoly market in an M/M/1 queueing system.

It is worth mentioning that there are two main differences between the queueing and spectrum sharing models developed in this paper and those in existing studies. One is that the previous queueing models usually consist of two queues, while we develop a four-queue system that can support two kinds of service priority for duopoly market. Compared with the two-queue system in a monopoly market, where only one PO provides two kinds of service priority [18], [21], our four-queue system supports the competition between two POs, and can be straightforwardly extended to solve the spectrum sharing among multiple POs. Also, compared with the two-queue system in a duopoly market, where each PO owns one queue [19], [20], our model takes heterogeneous user demand into consideration. The second difference lies in that the existing studies only considered single-band spectrum sharing in which each PO leases only one band to SUs. In contrast, we consider a generic multiband scenario in an M/M/s queueing system, where POs determine not only the band price, but the number of bands they are willing to rent.

WE can be viewed as the solution of an optimization problem that minimizes the integral of the delays over all network links. It has been widely studied in the area of transportation [23], [24], and more recently in wireless and CR networks [20], [25]. In traditional two-queue systems, two Wardrop principles are easy to be guaranteed as long as the users in both queues experience the equal cost. For the proposed queueing system with four and more queues, the search of the WE point would be more complicated, as not only the costs of the users joining queues of the same priority, but the costs of the users selecting different priority queues should be considered. For a multiqueue system, how to find out the equilibrium traffic with a minimum average cost has remained vastly unexplored.

III. Scenarios and Queueing Models

Consider a CR network consisting of two POs, multiple PUs, and SUs. The POs lease their bands to SUs either in an exclusive or in an opportunistic fashion. As some SUs might choose to use the same PO band, congestion can occur at that PO. In order to describe the congestion effect incurred when multiple SUs wish to share the same band, we employ a "virtual" queue system [26], which consists of SUs waiting to use that band. In such a queueing system, each PO band is viewed as a server, and the customers are SUs waiting to access the band.

In the following sections, we will discuss three kinds of duopoly markets: 1) single-band exclusive use; 2) single-band mixed use; and 3) multiband exclusive use, as shown in Fig. 1. In single-band market, each PO owns a single band of some fixed bandwidth. If the band is exclusively used by SUs, it is an exclusive use market. If one PO band is for exclusive use while the other is for opportunistic access, it is a mixed use market. In multiband market, each PO provides multiple narrow bands dedicated for SUs.

In single-band exclusive use scenario, the queueing system of each PO band consists of an HP queue and an LP queue and assumes a PRP rule, with which SUs in HP queue (also called HP SUs) have absolute priority over SUs in LP queue (LP SUs). It means that when an LP SU is using the band and an HP SU arrives, the transmission of LP SU is interrupted and the band is occupied by the HP SU. Once there are no more HP SUs in the system, the LP SU resumes its transmission at the point, where it was interrupted.

For opportunistic access model, as the PU has pre-emptive priority over SUs, SUs must return the occupied channel when the PU appears at the channel. In this way, the band usage among the PU and SUs follows a 3-*level* PRP rule, in which the PU has pre-emptive priority of the highest level, and HP SUs has higher pre-emptive priority over LP SUs. In multiband scenario, as each PO serves SUs with multiple bands, it is modeled as an M/M/s queueing system.

IV. Single-Band Exclusive Use Market

In this section, we first analyze the delay performance of SUs in a single-band exclusive use competition, then find out the WE traffic, and finally present a parallel pricing strategy.

A. PRP M/M/1 Queueing Model

We assume that the arrival processes of SUs on the queues of each PO band are Poisson. Further, assume that the arrival processes of SUs on each PO band are also Poisson. Let $\lambda_{i,j}$ be the traffic arrival rate of SUs, who join PO i's queue j (we use $j = 1, 2$ to represent HP and LP queue, respectively). Let λ_i represent the arrival rate of SUs who select PO i band, and λ be the total arrival rate of SUs traffic, then we have $\lambda_{1,1} + \lambda_{1,2} = \lambda_1$, $\lambda_{2,1} + \lambda_{2,2} = \lambda_2$, and $\lambda_1 + \lambda_2 = \lambda$. Let μ_i denote the service rate of PO i, i.e., the maximum achievable transmission rate of PO i band. To keep a stable queueing system, the total traffic admitted in the network cannot exceed its capacity. Namely, we have $\lambda < \mu_1 + \mu_2$.

Without loss of generality, we use a PRP M/M/1 queueing model to analyze the delay performance of SUs. The queueing delay refers to the sojourn time of the SU in the system from its arrival until its departure when it finishes transmission. Let $T_{i,1}(\lambda_{i,1})$ and $T_{i,2}(\lambda_{i,1}, \lambda_{i,2})$ represent the expected queueing delay of SUs who wait in PO i's HP and LP queue, respectively. Also, let $T_i(\lambda_{i,1}, \lambda_{i,2})$ be the expected queueing delay of the SUs that select PO i. Based on well known results of queueing theory [27], we have

$$T_{i,1}(\lambda_{i,1}) = \frac{1}{\mu_i - \lambda_{i,1}} \tag{1}$$

$$T_i(\lambda_{i,1}, \lambda_{i,2}) = \frac{1}{\mu_i - \lambda_{i,1} - \lambda_{i,2}} \tag{2}$$

$$T_i(\lambda_{i,1}, \lambda_{i,2}) = \frac{\lambda_{i,1} T_{i,1}(\lambda_{i,1}) + \lambda_{i,2} T_{i,2}(\lambda_{i,1}, \lambda_{i,2})}{\lambda_{i,1} + \lambda_{i,2}}. \tag{3}$$

By combining (1)–(3), we have

$$T_{i,2}(\lambda_{i,1}, \lambda_{i,2}) = \frac{\mu_i}{(\mu_i - \lambda_{i,1} - \lambda_{i,2})(\mu_i - \lambda_{i,1})}. \tag{4}$$

Let α be an average delay cost units per time unit, and $p_{i,j}$ be the admission price charged by PO i to SUs in queue j. Therefore, the overall cost of SUs, who choose PO i and wait in queue j, is defined as

$$C_{i,j} = \alpha \cdot T_{i,j} + p_{i,j}. \tag{5}$$

Therefore, the overall cost perceived by SUs choosing PO i is defined as

$$C_i = \alpha \cdot T_i + p_i. \tag{6}$$

For the sake of simple analysis, we introduce p_i to represent the average admission price payed by SUs choosing PO i. Similar to (3), we have

$$p_i = \frac{\lambda_{i,1} p_{i,1} + \lambda_{i,2} p_{i,2}}{\lambda_{i,1} + \lambda_{i,2}}. \tag{7}$$

Furthermore, assume that each SU obtains a reward or a service value of R units after being served. Then, the utility of the SU choosing queue j of PO i is expressed as

$$U_{i,j} = R - C_{i,j} = R - (\alpha \cdot T_{i,j} + p_{i,j}). \tag{8}$$

As the utility of the SU may be negative when the total cost exceeds the reward, we assume that the SU would join the system if its utility is non-negative. Or else it chooses to balk with a zero utility.

B. Wardrop Equilibrium Traffic

When an SU arrives at the system, it requires to make two decisions. One is to select the PO, i.e., the SU makes a cost evaluation and chooses the PO at which it experiences the minimum cost. The other is to select the queue, i.e., the SU chooses the desired queue according to its QoS demand. As each SU make the decision selfishly and independently, this process can be modeled as a noncooperative game. Also, we assume that the number of SUs is large enough, so that the demand of each SU is infinitesimal with respect to the overall demands. Thus, the decision-making process of SUs can be modeled as a nonatomic game. The stable outcome of a nonatomic game is a WE, which in this paper refers to an optimal SUs incoming traffic partition among POs' queues.

Proposition 1 (WE): Given admission price $p_{i,j}$, $i, j = 1, 2$, a traffic partition $\lambda^{\mathrm{WE}} = (\lambda_{1,1}^{\mathrm{WE}}, \lambda_{1,2}^{\mathrm{WE}}, \lambda_{2,1}^{\mathrm{WE}}, \lambda_{2,2}^{\mathrm{WE}})$ is a WE point

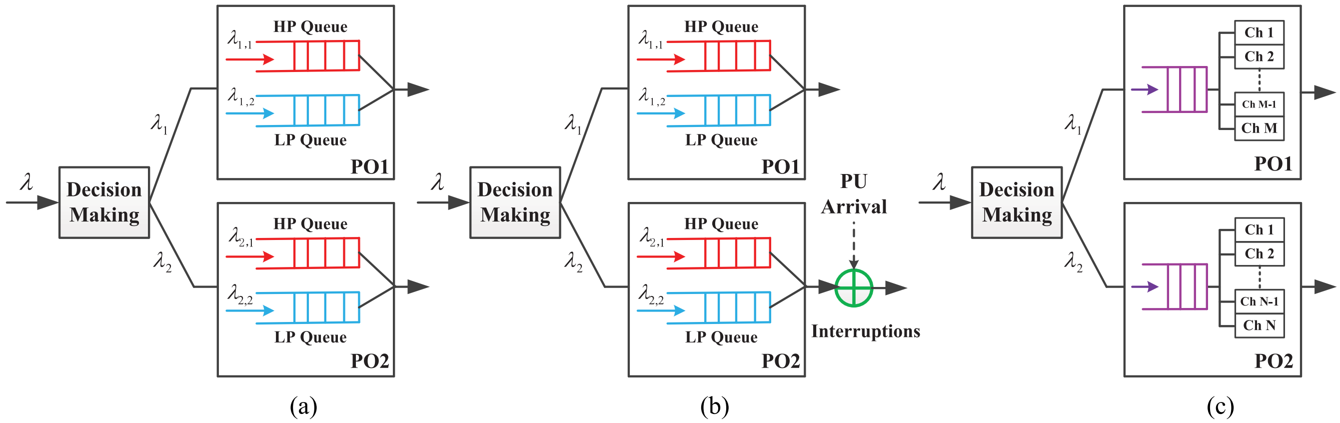


Fig. 1. Three kinds of duopoly markets and queueing models. (a) Single-band exclusive use with PRP M/M/1 queueing model, (b) single-band mixed use with PRP M/M/1 and 3-level PRP M/M/1 queueing model, and (c) multiband exclusive use with M/M/s queueing model.

if it satisfies

$$\lambda_{i,j}^{\mathrm{WE}}\left(R-\alpha T_{i,j}\left(\lambda_{i,j}^{\mathrm{WE}}\right)-p_{i,j}\right)=0$$
$$R-\alpha T_{i,j}\left(\lambda_{i,j}^{\mathrm{WE}}\right)-p_{i,j}\leq 0. \tag{9}$$

Proposition 2 [16]: At a WE, the incoming traffic satisfies two principles. The first is that the total costs experienced by SUs on all occupied spectrum are equal. The second is that the average cost of SUs is minimum.

Theorem 1: There exists a unique WE point for duopoly single-band exclusive use market.

Proof: At a WE point, the incoming traffic satisfies.

1) The overall cost perceived by SUs choosing PO 1 is equal to that choosing PO 2. Namely

$$\alpha\frac{1}{\mu_1-\lambda_{1,1}-\lambda_{1,2}}+p_1=\alpha\frac{1}{\mu_2-\lambda_{2,1}-\lambda_{2,2}}+p_2. \tag{10}$$

2) For SUs choosing any PO i, the overall cost perceived by SUs joining HP queue is equal to that joining LP queue. That is

$$\alpha\frac{1}{\mu_1-\lambda_{1,1}-\lambda_{1,2}}+p_{1,1}=\alpha\frac{\mu}{(\mu_1-\lambda_{1,1}-\lambda_{1,2})(\mu_1-\lambda_{1,1})}+p_{1,2} \tag{11}$$

and

$$\alpha\frac{1}{\mu_2-\lambda_{2,1}-\lambda_{2,2}}+p_{2,1}=\alpha\frac{\mu}{(\mu_2-\lambda_{2,1}-\lambda_{2,2})(\mu_2-\lambda_{2,1})}+p_{2,2}. \tag{12}$$

It is easy to show that there exists a unique set of $\lambda_{i,j}^{\mathrm{WE}}$, which satisfies (10)–(12), as well as $\sum_i\sum_j\lambda_{i,j}=\lambda$ with an arbitrarily original value of λ. ■

C. Parallel Pricing Strategy

The utilities (revenues) of PO 1 and PO 2 are, respectively, expressed as

$$\pi_1=(\lambda_{1,1}+\lambda_{1,2})p_1=\lambda_1 p_1 \tag{13}$$
$$\pi_2=(\lambda_{2,1}+\lambda_{2,2})p_2=\lambda_2 p_2. \tag{14}$$

As both POs allow SUs to use the spectrum exclusively, we consider a parallel pricing strategy, with which two POs set their prices simultaneously to guarantee the fairness. For this noncooperative game, we define an NE price set $\mathbf{P}^{\mathrm{NE}}=(p_1^{\mathrm{NE}},p_2^{\mathrm{NE}})$, at which no PO can increase its revenue by unilaterally deviating from this price.

The revenue maximization problems of the two POs are defined as

$$\max_{p_1\geq 0}\ \lambda_1 p_1 \quad \text{s.t. } \alpha T_1(\lambda_1)+p_1=\alpha T_2(\lambda_2)+p_2 \tag{15}$$
$$\max_{p_2\geq 0}\ \lambda_2 p_2 \quad \text{s.t. } \alpha T_1(\lambda_1)+p_1=\alpha T_2(\lambda_2)+p_2. \tag{16}$$

Theorem 2: The parallel pricing strategy has a unique NE point.

Proof: It is hard to directly find out the optimal solution of p_1 and p_2, so we first calculate the optimal λ_1 and λ_2 and obtain the equivalent maximization problems as

$$\max_{\lambda_1\geq 0}\ \lambda_1(\alpha T_2(\lambda_2)+p_2-\alpha T_1(\lambda_1)) \tag{17}$$
$$\max_{\lambda_2\geq 0}\ \lambda_2(\alpha T_1(\lambda_1)+p_1-\alpha T_2(\lambda_2)). \tag{18}$$

From (17), PO 1 maximizes the revenue by setting $d\pi_1/d\lambda_1=0$, and obtains

$$p_2=\alpha T_1(\lambda_1)-\alpha T_2(\lambda_2)+\alpha\lambda_1\left(\frac{dT_1(\lambda_1)}{d\lambda_1}-\frac{dT_2(\lambda-\lambda_1)}{d\lambda_1}\right). \tag{19}$$

Similarly, PO 2 maximizes the revenue by using $d\pi_2/d\lambda_2=0$, and has

$$p_1=\alpha T_2(\lambda_2)-\alpha T_1(\lambda_1)+\alpha\lambda_2\left(\frac{dT_2(\lambda_2)}{d\lambda_2}-\frac{dT_1(\lambda-\lambda_2)}{d\lambda_2}\right). \tag{20}$$

In addition, with the WE condition in (10), we have

$$p_1-p_2=\alpha T_2(\lambda_2)-\alpha T_1(\lambda_1). \tag{21}$$

Combining (19)–(21), we have

$$\frac{2\lambda_1-\lambda}{(\mu_1-\lambda_1)^2}+\frac{2\lambda_1-\lambda}{(\mu_2-\lambda+\lambda_1)^2}=\frac{1}{\mu_1-\lambda_1}-\frac{1}{\mu_2-\lambda+\lambda_1}. \tag{22}$$

TABLE I
EQUILIBRIUM ARRIVAL RATE AND EQUILIBRIUM PRICE WITH PARALLEL PRICING STRATEGY ($\mu_1 = 5, \mu_2 = 7, \alpha = 0.1$)

Arrival rate λ	Equilibrium arrival rate $(\lambda_{1,1}^{\mathrm{WE}}, \lambda_{1,2}^{\mathrm{WE}}, \lambda_{2,1}^{\mathrm{WE}}, \lambda_{2,2}^{\mathrm{WE}})$	Equilibrium price $(p_{1,1}^{\mathrm{NE}}, p_{1,2}^{\mathrm{NE}}, p_{2,1}^{\mathrm{NE}}, p_{2,2}^{\mathrm{NE}})$	PO revenue (π_1, π_2)
2	$(0.3164, 0.5178, 0.4721, 0.6937)$	$(0.0073, 0.0045, 0.0096, 0.0066)$	(0.004672,0.009093)
3	$(0.5154, 0.8189, 0.4721, 1.1936)$	$(0.0141, 0.0089, 0.0238, 0.0094)$	(0.014410,0.022487)
4	$(0.719, 1.1156, 0.9045, 1.2609)$	$(0.0235, 0.0151, 0.0260, 0.0186)$	(0.033757,0.046989)
5	$(0.9314, 1.4035, 1.1321, 1.533)$	$(0.0374, 0.0248, 0.0400, 0.0296)$	(0.069580,0.090613)
6	$(1.1521, 1.6833, 1.3681, 1.7965)$	$(0.0586, 0.0401, 0.0614, 0.0468)$	(0.134965,0.168040)
7	$(1.3827, 1.9534, 1.6141, 2.0498)$	$(0.0928, 0.0657, 0.0960, 0.0756)$	(0.25687,0.309966)

Equation (22) is a cubic equation of λ_1 that has only one real root as follows [29]:

$$\lambda_1 = \left(-\frac{q}{2} + \left(\left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3\right)^{\frac{1}{2}}\right)^{\frac{1}{3}} + \left(-\frac{q}{2} - \left(\left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3\right)^{\frac{1}{2}}\right)^{\frac{1}{3}} \tag{23}$$

where

$$p = \frac{5\mu_1^2 + 5\mu_2^2 + 9\lambda^2 + 26\mu_1\mu_2 - 18\mu_1\lambda - 18\mu_2\lambda}{12}$$
$$q = \frac{2\mu_1^3 - 6\mu_1^2\mu_2 + 6\mu_1\mu_2^2 - 2\mu_2^3 - 45\mu_1\mu_2\lambda}{27}. \tag{24}$$

After finding a unique solution of λ_1, we can determine the optimal λ_2 by using $\lambda_2 = \lambda - \lambda_1$. Finally, we obtain the unique NE price p_1^{NE} and p_2^{NE} by (19) and (20). ■

With the optimal price p_i^{NE}, each PO i further determines its optimal charges $p_{i,1}^{\mathrm{NE}}$ and $p_{i,2}^{\mathrm{NE}}$ for entering its two queues. Following Proposition 1, the overall cost of HP SUs equals to that of LP SUs who choose the same PO band. Namely

$$\alpha T_{i,1}(\lambda_{i,1}) + p_{i,1} = \alpha T_{i,2}(\lambda_{i,1}, \lambda_{i,2}) + p_{i,2}. \tag{25}$$

Define $\lambda_i C_i = \sum_{j=1}^{2} \lambda_{i,j} C_{i,j}$, we have

$$\alpha T_i(\lambda_{i,1}, \lambda_{i,2}) + p_i = \alpha T_{ij}(\lambda_{i,j}) + p_{i,j}. \tag{26}$$

Therefore, PO i maximizes the revenue of HP queue by

$$\max_{p_{i,1}\geq 0} \ \lambda_{i,1}p_{i,1}$$
$$\text{s.t.} \ (\lambda_{i,1} + \lambda_{i,2})p_i = \lambda_{i,1}p_{i,1} + \lambda_{i,2}p_{i,2}. \tag{27}$$

Similarly, PO i sets the admission price of LP queue in the same way and obtain

$$\max_{p_{i,2}\geq 0} \ \lambda_{i,2}p_{i,2}$$
$$\text{s.t.} \ (\lambda_{i,1} + \lambda_{i,2})p_i = \lambda_{i,1}p_{i,1} + \lambda_{i,2}p_{i,2}. \tag{28}$$

By relaxing the constraint and applying the WE conditions in (11) and (12), problem (27) and (28) can be rewritten as

$$\lambda_{i,1}p_{i,1} = (\lambda_i - \lambda_{i,1})(C_i - \alpha T_{i,2}(\lambda_{i,1}))$$
$$\text{s.t.} \ C_i = \alpha T_{i,1}(\lambda_{i,1}) + p_{i,1}. \tag{29}$$

It is easy to find the optimal $\lambda_{i,1}^{\mathrm{WE}}$ and $p_{i,1}^{\mathrm{NE}}$ from (29). Thereafter, we have $\lambda_{i,2}^{\mathrm{WE}} = \lambda_i^{\mathrm{WE}} - \lambda_{i,1}^{\mathrm{WE}}$ and $p_{i,2}^{\mathrm{NE}} = C_i - \alpha T_{i,2}(\lambda_{i,2}^{\mathrm{WE}})$.

D. *Price of Anarchy*

We now introduce the metric of the price of anarchy (PoA) to evaluate the efficiency of the parallel pricing strategy.

First, we give out the definition of the social welfare. The social welfare in this paper is defined as the summation of POs' revenues and SUs' utilities. Namely

$$S = \sum_i \sum_j \lambda_{i,j}p_{i,j} + \sum_i \sum_j R\lambda_{i,j} - \sum_i \sum_j \lambda_{i,j}(\alpha T_{i,j}(\lambda_{i,j}) + p_{i,j}) = R\lambda - \alpha \sum_i \sum_j \lambda_{i,j}T_{i,j}(\lambda_{i,j}). \tag{30}$$

The social welfare maximization problem is then defined as

$$\max \ S$$
$$\text{s.t.} \ \lambda = \textstyle\sum_i \sum_j \lambda_{i,j}$$
$$\lambda_{i,j} \geq 0 \quad \text{for } i = 1, 2; j = 1, 2. \tag{31}$$

The objective function is continuous and the constraint set is compact, thus, there always exists a solution $S^{\max}$ for (31). As in [28], we define the PoA as the ratio between the social welfare S^{NE} achieved at the NE point and the maximum social welfare $S^{\max}$ obtained from (31). That is,

$$\mathrm{PoA} = \frac{S^{\mathrm{NE}}}{S^{\max}}. \tag{32}$$

Since $S^{\mathrm{NE}} \leq S^{\max}$, we have $0 < \mathrm{PoA} \leq 1$. Therefore, the closer the PoA to 1, the closer the achieved equilibrium to the maximum $S^{\max}$.

E. *Numerical Results*

We now analyze the numerical results and show the equilibrium performance of the exclusive use market with parallel pricing strategy. Table I displays the equilibrium behavior of POs and SUs with different arrival rates, where the service rates of two POs are set to 6 and 7, respectively, and the overall arrival rate λ of SUs ranges in [2, 7]. It is shown the existence of the equilibrium arrival rates of SUs on each queue, and the equilibrium admission prices of POs under different λ. Also, the equilibrium arrival rates and prices increase with λ. It is observed that, by offering a service rate μ_2 larger than μ_1, PO 2 attracts more SUs and then achieves more revenue.

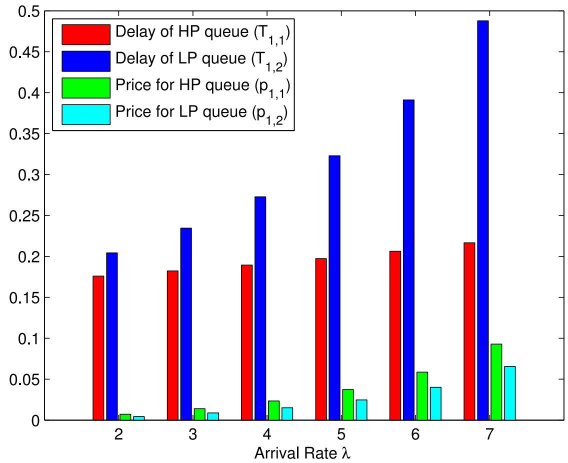


Fig. 2. Delay and charge of PO 1 for different queue.

Fig. 2 compares the average queueing delay and the charge of PO 1 for different queues. It is seen that the delay of HP queue is obviously smaller than that of LP queue. In return, the charge for HP queue is much higher than for LP queue. Moreover, with the increase of the arrival rate λ, the delay of HP queue grows slightly while the delay of LP queue increases significantly. Since compared with the LP queue, the HP queue has pre-emptive priority, its delay performance is not easily impacted by the increase of the traffic.

Fig. 3 shows the impact of the service rate μ_1 on the average queueing delay and the admission price, where the service rate μ_2 is fixed at 7, and the arrival rate λ is set to 5. It is seen that the average queueing delay of both POs decrease with μ_1, but its effect on PO 1 is much stronger. The reason lies in that with the increasing service rate of PO 1, PO 1's queueing delay would be directly reduce. On the other hand, some SUs once selecting PO 2 would turn to PO 1, resulting in the decrease of PO 2's queueing delay. Further, as μ_1 approaches to μ_2, the differences of queueing delay and admission charge of these two POs gradually shrink. When μ_1 equals to μ_2, by using the parallel pricing strategy, the services offered by two POs and the charges from them would be the same, which consequently results in the same delay performance of two POs.

We now measure the efficiency of the equilibrium reached by the parallel pricing strategy. Fig. 4 shows the values of the PoA achieved under different service rate settings. It is found that when both POs offer the same service rate, the PoA is equal to 1, which means that the equilibrium reaches the global efficient point. This is because that as μ_1 is equal to μ_2, as shown in Fig. 3, the delay and charge of both POs are the same. Therefore, the aggregate cost of SUs joining two POs is minimized, which results in the global optimum. If we gradually increase the gap $|\mu_1 - \mu_2|$ between the service rates of two POs, the PoA would slightly decrease. For example, when $\mu_1 = 8$ and $\mu_2 = 5$, there is an average efficiency loss of 0.5% with respect to the social optimum.

V. Single-Band Mixed Use Market

This section considers a single-band mixed use market. First, we derive the average queueing delay of SUs in a 3-level PRP M/M/1 queueing system, and then present a leader–follower pricing strategy, in which the competition between two POs is modeled as a two-stage Stackelberg game.

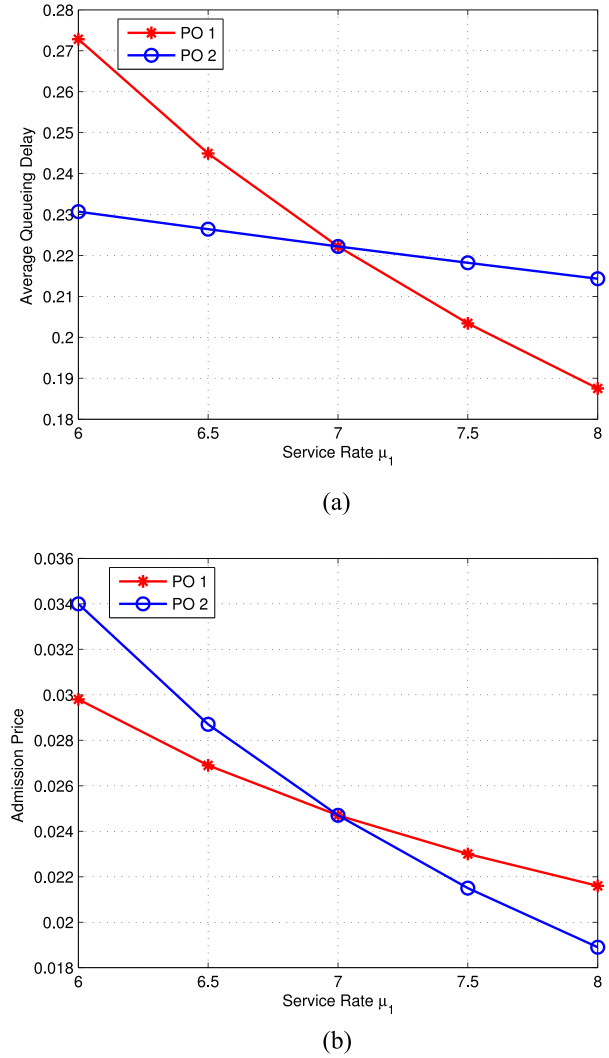


Fig. 3. Impacts of μ_1 on the delay and charge of two POs.

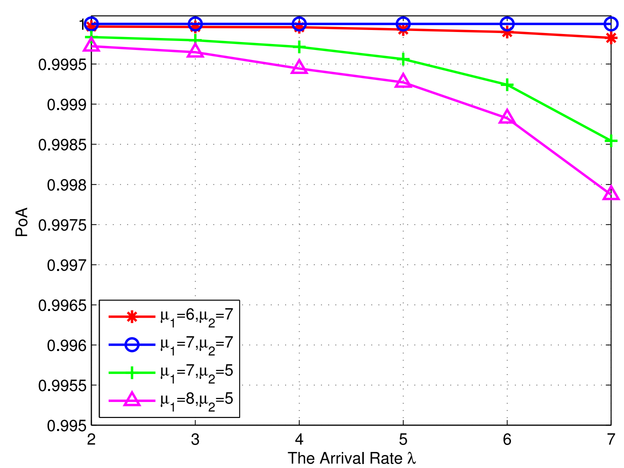


Fig. 4. PoA versus service rate.

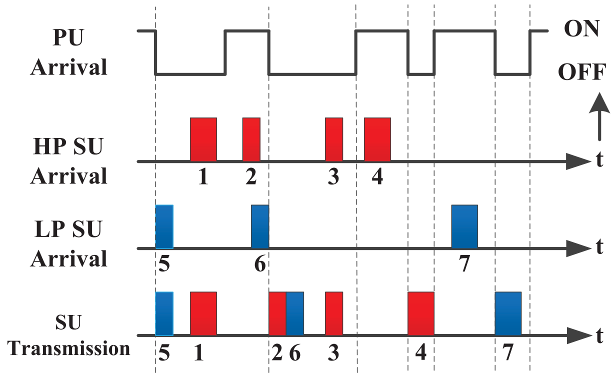


Fig. 5. Transmission of four HP SUs and three LP SUs, who have different arrival time and band occupation period.

A. 3-Level PRP M/M/1 Queueing Model

Assume that the traffic pattern of the PU is an ON/OFF exponential model. For other traffic patterns, they can be analyzed in a similar way. The ON/OFF model is composed of two alternating phases: 1) an active phase (ON) in which the PU uses the band licensed to it and 2) an inactive phase (OFF) during which the PU does not use the channel. Assume that the ON and OFF period are independent and exponentially distributed with mean $1/\eta$ and $1/\xi$, respectively. For SUs performing spectrum sensing, we assume an accurate detection. Fig. 5 shows the transmission of seven SUs in OFF phase.

Before we model the competition of two POs, we first analyze the delay performance of SUs in opportunistic access on the basis of 3-level PRP M/M/1 queueing model.

1) Queueing Delay of SUs in HP Queue: As SUs can use the band only when the PU is in an OFF phase, the service rate of SUs choosing PO i band is defined as $\bar{\mu}_i = (\eta/\eta+\xi)\mu_i$, and the average service time for each SU is $D = 1/\bar{\mu}_i$. Define $T_{i,1}(\lambda_{i,1})$ as the expected queueing delay of SUs selecting HP queue, we have

$$T_{i,1}(\lambda_{i,1}) = E(W_{i,1}) + D \tag{33}$$

where $W_{i,1}$ represents the waiting time of the SU in queue 1. It refers to the time during which the SU waits in the queue before using the band. Generally, $W_{i,1}$ is composed of three parts.

1) The remaining service time T_r of the SU that is using the band at the time of arrival.
2) The total service time T_w required for the SUs who are queueing ahead.
3) While waiting in the queue, the SU must also wait for interruptions from the PU, say T_p.

By taking expectations, we obtain

$$E(W_{i,1}) = E(T_r) + E(T_w) + E(T_p). \tag{34}$$

Since one SU's service time is $1/\bar{\mu}_i$, we have $E(T_r) = (\lambda_{i,1}/\bar{\mu}_i)(1/\bar{\mu}_i)$. Assume that there are on average L_q SUs ahead of the arriving SU, then T_w is given by

$$E(T_w) = \frac{L_q}{\bar{\mu}_i} = \frac{\lambda_{i,1}E(W_{i,1})}{\bar{\mu}_i} \tag{35}$$

where $L_q = \lambda_{i,1}E(W_{i,1})$ is achieved by Little's law [22].

Furthermore, the steady-state probability of the PU arriving when the SU waits in the queue is defined as: $P_{\text{on}} = (\eta/\eta+\xi)$. Thus, the service time of the arriving PU is given by

$$E(T_p) = P_{\text{on}} \cdot \frac{1}{\eta} = \frac{1}{\eta+\xi}. \tag{36}$$

Combining the above expressions, we obtain

$$\begin{aligned} E(W_{i,1}) &= \frac{\lambda_{i,1}}{\bar{\mu}_i}\frac{1}{\bar{\mu}_i} + \frac{\lambda_{i,1}E(W_{i,1})}{\bar{\mu}_i} + \frac{1}{\eta+\xi} \\ \Rightarrow E(W_{i,1}) &= \frac{\bar{\mu}_i^2\frac{1}{\eta+\xi} + \lambda_{i,1}}{\bar{\mu}_i(\bar{\mu}_i - \lambda_{i,1})}. \end{aligned} \tag{37}$$

Finally, by incorporating (37) into (33), we have

$$T_{i,1}(\lambda_{i,1}) = \frac{\bar{\mu}_i^2\frac{1}{\eta+\xi} + \lambda_{i,1}}{\bar{\mu}_i(\bar{\mu}_i - \lambda_{i,1})} + \frac{1}{\bar{\mu}_i} = \frac{1+\frac{\mu_i\eta}{(\eta+\xi)^2}}{\bar{\mu}_i - \lambda_{i,1}} = \frac{\kappa}{\bar{\mu}_i - \lambda_{2,1}} \tag{38}$$

where $\kappa = 1 + [(\mu_i\eta)/((\eta+\xi)^2)]$.

2) Queueing Delay of SUs in LP Queue: Similar to the formulations of (1)–(3), we have

$$T_{i,2}(\lambda_{i,1}, \lambda_{i,2}) = \frac{\kappa\bar{\mu}_i}{(\bar{\mu}_i - \lambda_{i,1} - \lambda_{i,2})(\bar{\mu}_i - \lambda_{i,1})}. \tag{39}$$

Definition 1: For a 3-level PRP M/M/1 queueing model, the expected queueing delay of HP SUs that choose any PO i band is given by

$$T_{i,1}(\lambda_{i,1}) = \frac{\kappa}{\bar{\mu}_i - \lambda_{2,1}} \tag{40}$$

and the expected queueing delay of LP SUs choosing any PO i band is given by

$$T_{i,2}(\lambda_{i,1}, \lambda_{i,2}) = \frac{\kappa\bar{\mu}_i}{(\bar{\mu}_i - \lambda_{i,1} - \lambda_{i,2})(\bar{\mu}_i - \lambda_{i,1})}. \tag{41}$$

B. Leader–Follower Pricing Strategy

Assume a single-band mixed use market, where PO 1 band is for exclusive use and PO 2 band is for opportunistic access, then the expected queueing delay of two POs are listed as follows:

$$\begin{aligned} T_{1,1}(\lambda_{1,1}) &= \frac{1}{\mu_1 - \lambda_{1,1}},\ T_{2,1}(\lambda_{2,1}) = \frac{\kappa}{\bar{\mu}_2 - \lambda_{2,1}} \\ T_{1,2}(\lambda_{1,1}, \lambda_{1,2}) &= \frac{\mu_1}{(\mu_1 - \lambda_{1,1} - \lambda_{1,2})(\mu_1 - \lambda_{1,1})} \\ T_{2,2}(\lambda_{2,1}, \lambda_{2,2}) &= \frac{\kappa\bar{\mu}_2}{(\bar{\mu}_2 - \lambda_{2,1} - \lambda_{2,2})(\bar{\mu}_2 - \lambda_{2,1})} \\ T_1(\lambda_{1,1}, \lambda_{1,2}) &= \frac{1}{\mu_1 - \lambda_{1,1} - \lambda_{1,1}} \\ T_2(\lambda_{2,1}, \lambda_{2,2}) &= \frac{\kappa}{\bar{\mu}_2 - \lambda_{2,1} - \lambda_{2,2}}. \end{aligned} \tag{42}$$

As a matter of fact, the parallel pricing strategy can also be used in mixed use market to determine the admission prices of two POs. Considering PO 1 provides the band for dedicated use, we model the competition of two POs as a two-stage Stackelberg game [32]: PO 1 is the Stackelberg leader, which sets price to maximize the revenue, by taking into account that its decision will affect the price selected by the Stackelberg follower (PO 2). The outcome of this game is an NE price set $\mathbf{P}^{\mathrm{NE}} = (p_1^{\mathrm{NE}}, p_2^{\mathrm{NE}})$. In a two-stage Stackelberg game, the leader PO 1 sets its price p_1 in the first stage, and the follower PO 2 decides its price p_2 based on p_1 in the second stage. A typical solution to such a dynamic game is to use backward induction approach, with which the optimal price of PO 2 will be determined first.

In stage II, PO 2 maximizes its revenue by

$$\max_{p_2 \geq 0} \ \pi_2 = \lambda_2 p_2$$
$$\text{s.t.} \ \alpha T_1(\lambda_1) + p_1 = \alpha T_2(\lambda_2) + p_2. \tag{43}$$

Similar to the parallel pricing strategy, we change the variable p_2 to λ_2. By taking the derivative of π_2 in (43) with respect to λ_2, and letting $d\pi_2/d\lambda_2 = 0$, we obtain the equilibrium price p_1 as a function of λ_2

$$p_1 = \frac{\alpha\kappa\bar{\mu}_2}{(\bar{\mu}_2 - \lambda + \lambda_1)^2} + \frac{\alpha(\lambda - \mu_1)}{(\mu_1 - \lambda_1)^2}. \tag{44}$$

Thereafter, in stage I, combining (44), PO 1 maximizes its revenue by

$$\max_{\lambda_1 \geq 0} \ \pi_1 = \lambda_1\left(\frac{\alpha\kappa\bar{\mu}_2}{(\bar{\mu}_2 - \lambda + \lambda_1)^2} + \frac{\alpha(\lambda - \mu_1)}{(\mu_1 - \lambda_1)^2}\right)$$
$$\text{s.t.} \ \lambda_1 + \lambda_2 = \lambda. \tag{45}$$

By setting $d\pi_1/\partial\lambda_1 = 0$, we achieve

$$\frac{\kappa\bar{\mu}_2(\bar{\mu}_2 - \lambda - \lambda_1)}{(\bar{\mu}_2 - \lambda + \lambda_1)^3} - \frac{(\mu_1 - \lambda)(\mu_1 + \lambda_1)}{(\mu_1 - \lambda_1)^3} = 0. \tag{46}$$

The root of (46), i.e., the equilibrium arrival rate λ_1^{WE}, can be found by using root-finding algorithms, such as the bisection method with logarithmic complexity [30]. Finally, we can find out the NE price p_1^{NE} and p_2^{NE} as follows:

$$p_1^{\mathrm{NE}} = \frac{\alpha\kappa\bar{\mu}_2}{\left(\bar{\mu}_2 - \lambda + \lambda_1^{\mathrm{WE}}\right)^2} + \frac{\alpha(\lambda - \mu_1)}{\left(\mu_1 - \lambda_1^{\mathrm{WE}}\right)^2}$$
$$p_2^{\mathrm{NE}} = \alpha T_1\left(\lambda_1^{\mathrm{WE}}\right) + p_1^{\mathrm{NE}} - \alpha T_2\left(\lambda_2^{\mathrm{WE}}\right). \tag{47}$$

C. Numerical Results

We now evaluate the performance of the mixed use competition with leader-follow pricing strategy. For PO 2 which allows SUs to opportunistically share the spectrum with the PU, Fig. 6 shows the effect of the PU traffic pattern on the average queueing delay of SUs joining PO 2. Here, we define $\mu_1 = 7$, $\mu_2 = 7$, and $\alpha = 0.1$. It is seen that with the increase of η (the decrease of PU's ON period), or the decrease of ξ (the increase of PU's OFF period), the SUs selecting PO 2 obtain more time to use the spectrum, thus leading to a lower queueing delay. Moreover, for a certain PU traffic model, the queueing delay of the SUs increases with the arrival rate λ.

Fig. 7 compares the average queueing delay and admission price of two POs, where PO 1 offers exclusive spectrum use and PO 2 provides opportunistic spectrum access. Here, we have $\mu_1 = 7$, $\mu_2 = 7$, $\alpha = 0.1$, $\eta = 0.25$, and $\xi = 0.05$. As seen in Fig. 7(a), the average delay of SUs selecting PO 2 is much higher than that selecting PO 1. This is due to the fact that SUs which choose PO 2 have to share the spectrum with the PU, thus obtaining less time to use the spectrum. Also, compared with two LP queues, the delay of the two HP queues grow at a very slow speed with the arrival rate λ. Since

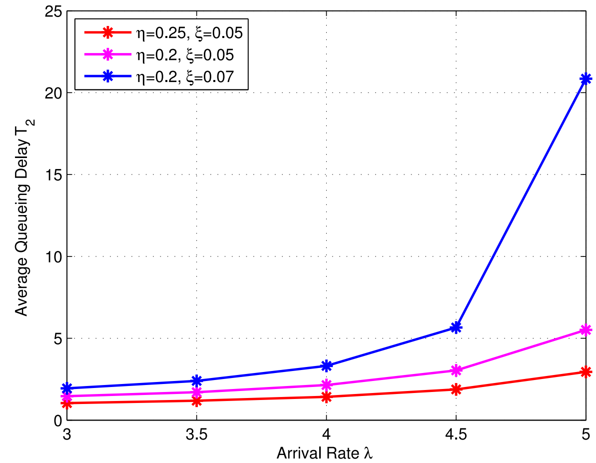
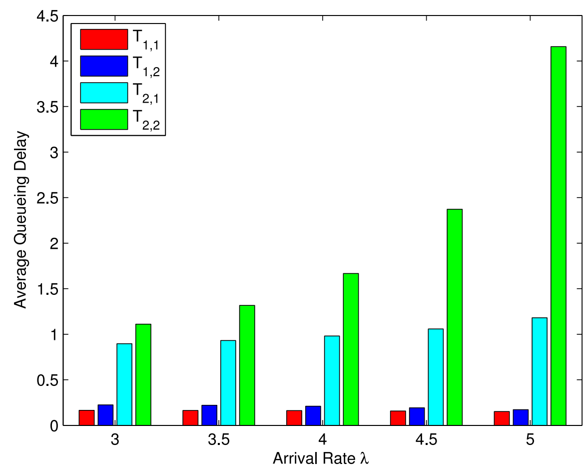
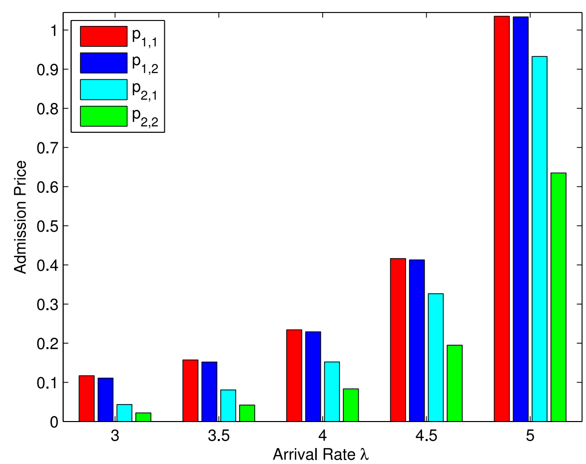


Fig. 6. Impact of PU traffic on average queueing delay.



(a)



(b)

Fig. 7. Exclusive use model versus opportunistic access model.

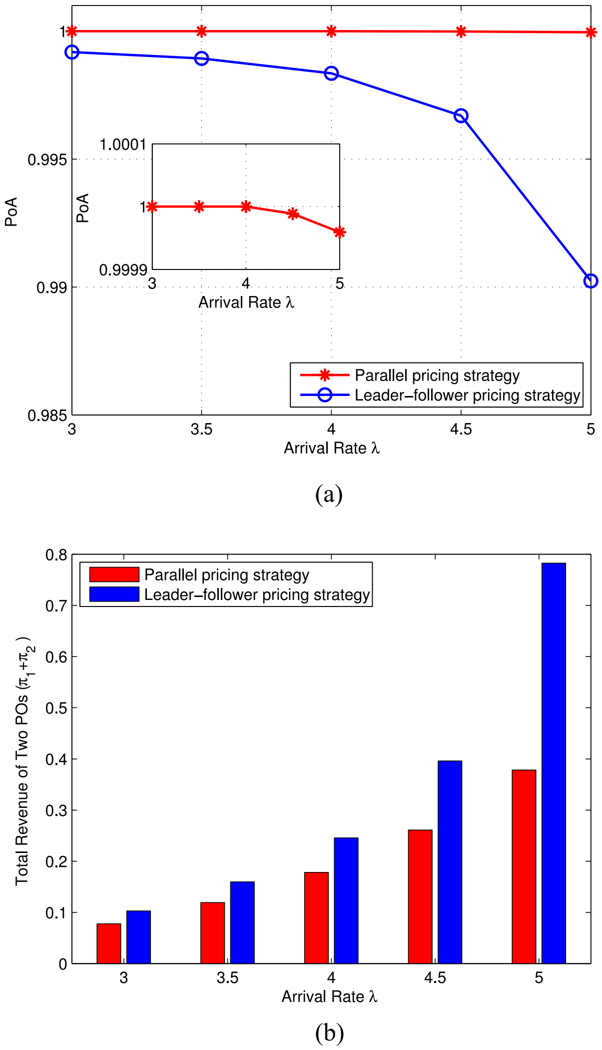


Fig. 8. Parallel pricing versus leader–follower pricing.

low latency means high payment, the inverse results for POs' charge can be found in Fig. 7(b).

Fig. 8 compares the performance of the mixed use market under the parallel and leader–follower pricing strategy, where the parallel (leader–follower) pricing strategy means that both POs adopt the parallel (leader–follower) pricing strategy. Here, we have $R = 2$, $\mu_1 = 5$, $\mu_2 = 7$, $\alpha = 0.1$, $\eta = 0.8$, and $\xi = 0.1$. It is observed in Fig. 8(a) that the PoA achieved by the parallel pricing strategy is very close to 1, which means that the equilibrium reached by this strategy nearly gets the globally efficient point. For the leader–follower strategy, the PoA decreases with the arrival rate λ, and there is an average efficiency loss of 0.6% with respect to the social optimum. Although the PoA performance of the parallel strategy is better than that of the follower–leader strategy, the follower–leader strategy outperforms the parallel strategy in terms of the revenue of two POs. As seen in Fig. 8(b), the total revenue of two POs is obviously higher with the follower–leader strategy. Therefore, The selfish POs are inclined to choose the leader–follower strategy for more revenue.

VI. Multiband Exclusive Use Market

In this section, we extend the study to a multiband exclusive use market, where each PO has multiple idle bands that can be leased to SUs for exclusive use. For maximizing the utility, the POs have to determine not only the admission price, but the number of the bands they are willing to rent. Here, we assume that the bands owned by the same PO are identical. Note that the difference of multiband mixed use market and exclusive use market lies in that in the mixed use market, there exists one PO who provides opportunistic access for the SUs. The delay performance of the SUs in opportunistic access can be analyzed as in single-band mixed use shown in Section V-A.

A. Problem Formulation

Similarly, assume that the incoming traffic from SUs who select PO i band arrives with a Poisson process of rate λ_i, and the overall arrival rate of SUs satisfies $\lambda_1 + \lambda_2 = \lambda$. Since each PO i serve SUs with multiple bands, the system can be represented by an M/M/s queueing model, where s denotes the number of servers (bands). Let μ_i be the service rate of PO i, $s_i^{\max}$ be the maximum number of idle bands PO i has, and s_i be the number of bands PO i will rent, then the expected queueing delay of SUs choosing PO i is given by [27]

$$T_i = \frac{\left(\frac{\lambda_i}{\mu_i}\right)^{s_i}\left[\sum_{k=0}^{s_i-1}\frac{\left(\frac{\lambda_i}{\mu_i}\right)^k}{k!} + \frac{\left(\frac{\lambda_i}{\mu_i}\right)^{s_i}}{s_i!}\frac{\mu_i s_i}{\mu_i s_i - \lambda_i}\right]^{-1}}{\mu_i s_i s_i!\left(1-\frac{\lambda_i}{\mu_i s_i}\right)^2}. \quad (48)$$

The aggregate cost of SUs choosing PO i is defined as

$$C_i = \alpha T_i(\lambda_i, s_i) + p_i \quad (49)$$

where p_i is the admission price set by PO i.

Since the POs aim at maximizing their revenue, while the SUs try to minimize their overall cost, the optimization problem is formulated as

$$\begin{aligned}
&\max \ \lambda_1 p_1\\
&\max \ \lambda_2 p_2\\
&\min \ \lambda_1(\alpha T_1(\lambda_1, s_1) + p_1) + \lambda_2(\alpha T_2(\lambda_2, s_2) + p_2)\\
&\ \text{s.t.} \ \lambda_1 + \lambda_2 = \lambda\\
&\qquad s_1 \in \left\{1, 2, \ldots, s_1^{\max}\right\}, s_2 \in \left\{1, 2, \ldots, s_2^{\max}\right\}\\
&\qquad \lambda_1 < \mu_1, \lambda_2 < \mu_2.
\end{aligned} \quad (50)$$

For this multiobjective programming problem, we use a weighting method [31] to combine three objective functions into a single objective function. Namely

$$\begin{aligned}
\mathbf{P1}: \ \max \ &\{\beta_1\lambda_1 p_1 + \beta_2\lambda_2 p_2 - \beta_3(\lambda_1(\alpha T_1(\lambda_1, s_1) + p_1)\\
&\quad + \lambda_2(\alpha T_2(\lambda_2, s_2) + p_2))\}\\
\text{s.t.} \ &\lambda_1 + \lambda_2 = \lambda\\
&s_1 \in \left\{1, 2, \ldots, s_1^{\max}\right\}, s_2 \in \left\{1, 2, \ldots, s_2^{\max}\right\}\\
&\lambda_1 < \mu_1, \ \lambda_2 < \mu_2
\end{aligned} \quad (51)$$

where $0 \le \beta_i \le 1$, and $\sum_i \beta_i = 1$.

The problem in (51) is a mix integer linear programming (MILP) problem that is in general NP-hard. To prove it, we first define our decision-making scheme for SUs in CR networks as DMS problem, which can be formally defined as follows.

Definition 2 (Decision-Making Scheme): Given a CR network consisting of two POs, (which could also be extended to m POs), a set S of SUs, in a multiband exclusive use competition, each SU s_k will choose a PO i for service, with a corresponding delay T_i^k, and a price payed to PO i as p_i. The DMS problem is to determine a scheduling scheme satisfying M/M/s queueing model at each PO, to maximize the total revenue collected to POs and minimize the total delay and payment for each SU simultaneously.

To prove the NP-completeness of DMS problem, we first convert this problem into a decision version, where we have a price constraint P and a delay constraint D, such that the total price is no less than P and the total delay is no more than D. Easily checking that given a scheduling scheme, we can check its correctness within polynomial time, so DMS is an NP problem.

Next, we prove the NP-completeness of DMS problem by a polynomial-time many-to-one reduction from scheduling to minimize weighted completion time (SMWCT) problem, whose definition is shown as follows (converted into decision version already).

Definition 3 (SMWCT): Give a set T of tasks, number m processors, for each task $t_i \in T$ a length l_i and a weight w_i, and a positive integer K, the SMWCT problem tries to find an m-processor schedule σ for T such that the sum, over all $t_i \in T$, of $(\sigma(t_i) + l_i) \cdot w_i$ is no more than K.

SMWCT problem is proved to be NP-complete (even with $m = 2$) (refer to A5.1–SS13 in [36]). Now, let us prove the NP-completeness of DMS.

Theorem 3: SMWCT $\leq_m^p$ DMS.

Proof: Given an instance of SMWCT problem (with $m = 2$), we can easily convert it into an instance of DMS problem polynomially. For each task t_i, we convert it into an SU s_k, the length l_i is converted into processing time of SU, and the weight w_i is converted into the cost satisfying M/M/s queueing model at each PO, which is waiting function related to the pricing policy. Then assume the price paid by each SU is unique as $(P/|T|)$, and the delay constraint D is equal to K. Respectively, finding a 2-processor schedule σ to satisfy SMWCT problem is equivalent as finding a scheme to each SU such that the overall price is no less than $(P/|T|) \cdot |T| = P$, and the overall delay is no more than $D = K$. ■

To successfully solve (51), we use decomposition approach and propose a two-level optimization procedure as follows:

$$\begin{aligned}\textbf{P1a}:\ &\max\ \{\beta_1\lambda_1 p_1 + \beta_2\lambda_2 p_2 - \beta_3(\lambda_1(\alpha T_1(\lambda_1, s_1) + p_1)\\ &\qquad + \lambda_2(\alpha T_2(\lambda_2, s_2) + p_2))\}\\ &\text{s.t.}\ \ s_1 \in \left\{1, 2, \ldots, s_1^{\max}\right\},\ s_2 \in \left\{1, 2, \ldots, s_2^{\max}\right\}\end{aligned} \tag{52}$$

$$\begin{aligned}\textbf{P1b}:\ &\max\ \{\beta_1\lambda_1 p_1 + \beta_2\lambda_2 p_2\}\\ &\text{s.t.}\ \ \alpha T_1(\lambda_1, s_1) + p_1 = \alpha T_2(\lambda_2, s_2) + p_2\\ &\qquad \lambda_1 + \lambda_2 = \lambda\\ &\qquad \lambda_1 < \mu_1, \lambda_2 < \mu_2.\end{aligned} \tag{53}$$

It is noted that compared with the objective function in (51), the third term (for minimizing the aggregate cost of SUs) is taken away from (53), as this objective is guaranteed by the WE principle shown in the first constraint of (53). Further, we make an inverse transformation, and rewrite P1b into a multiobjective optimization problem P1b′

$$\begin{aligned}\textbf{P1b}':\ &\max\ \ \lambda_1 p_1\\ &\max\ \ \lambda_2 p_2\\ &\text{s.t.}\ \ \alpha T_1(\lambda_1, s_1) + p_1 = \alpha T_2(\lambda_2, s_2) + p_2\\ &\qquad \lambda_1 + \lambda_2 = \lambda\\ &\qquad \lambda_1 < \mu_1,\ \lambda_2 < \mu_2.\end{aligned} \tag{54}$$

Clearly, P1b′ is a pricing problem where the number of bands to be leased is given. It can be directly solved by the proposed parallel or leader–follower pricing strategy. Given the available band numbers $\mathbf{s}$, the low level optimization procedure P1b′ is first implemented to find locally optimal price $\mathbf{p}$ and arrival rate λ, thereafter, the high level optimization procedure P1a is executed for updating $\mathbf{s}$. These two optimization steps alternately continue until P1a converges. It is worth mentioning that by using such a decomposition, the single-band scenario discussed previously becomes a special case of the multiband scenario, where P1a is neglected and P1b′ is solved by setting $s_1 = 1$ and $s_2 = 1$. In the following, we will propose a GPSO algorithm to effectively solve problem P1a.

B. Global Particle Swarm Optimization Solution

P1a is an IP problem. To find the solution in polynomial time, heuristic algorithms, such as particle swarm optimization (PSO) [33] can be utilized. PSO is characterized as easy to implement and computationally efficient when compared with other heuristic techniques, such as genetic algorithms.

PSO is a stochastic population-based optimization technique developed in 1995. PSO is initialized with a set of particles (also known as a swarm), which travel in the search space to find the global optimum of a fitness function. Each particle iteratively adjusts its velocity based on the experiences of itself and other particles. With the increase of the number of iterations, the swarm converge to the optimal position required for the global optimum.

Let D represent the dimension of the search space, $\mathbf{x}_i = (x_{i,1}, x_{i,2}, \ldots, x_{i,D})$ and $\mathbf{v}_i = (v_{i,1}, v_{i,2}, \ldots, v_{i,D})$ be the position and velocity of the ith particle, respectively. At each iteration, the velocity of the ith particle is updated as [34]

$$\begin{aligned}\mathbf{v}_i(t+1) = \omega\mathbf{v}_i(t) + c_1 r_1\left(\mathbf{p}_i^{\text{best}}(t) - \mathbf{x}_i(t)\right)\\ + c_2 r_2\left(\mathbf{g}_i^{\text{best}}(t) - \mathbf{x}_i(t)\right)\end{aligned} \tag{55}$$

where ω is an inertial weight that controls the local/global search scope, $\mathbf{p}_i^{\text{best}}$ represents the best previous position found by the ith particle, and $\mathbf{g}_i^{\text{best}}$ denotes the best position found by the swarm. c_1 and c_2 are, respectively, the local and global acceleration factors, and r_1 and r_2 are random values uniformly distributed in [0, 1]. The position of the ith particle is subsequently updated as

$$\mathbf{x}_i(t+1) = \mathbf{x}_i(t) + \mathbf{v}_i(t+1). \tag{56}$$

Meanwhile, the best previous position $\mathbf{p}_i^{\text{best}}(t+1)$ is computed by

$$\mathbf{p}_i^{\text{best}}(t+1) = \begin{cases} \mathbf{p}_i^{\text{best}}(t), & \text{if } f(\mathbf{x}_i(t+1)) \le f\left(\mathbf{p}_i^{\text{best}}(t)\right) \\ \mathbf{x}_i(t+1), & \text{otherwise} \end{cases} \tag{57}$$

where $f(\cdot)$ is the fitness function reflecting the quality of the solutions. The global best position $\mathbf{g}_i^{\text{best}}$ is given by

$$\mathbf{g}_i^{\text{best}}(t+1) = \arg\max_{\mathbf{p}_i^{\text{best}}} f\left(\mathbf{p}_i^{\text{best}}(t)\right). \tag{58}$$

One main issue of the PSO and other stochastic search algorithms is that the swarm may prematurely converge to a local optimum. Since the position and velocity of the particles are randomly initialized, they may be prevented from effective search for the global optimal and move toward the local optimum. In this case, the particles need to jump out and explore other possible search directions for better results. Therefore, when the fitness value of all the particles are concentrated, we can change the velocity with a random factor to explore other search directions.

That is, if $|\overline{f(t)} - f_i(\mathbf{g}_i^{\text{best}}(t))| \le f_{\text{th}}$, we have

$$\mathbf{v}_i(t+1) = \Big\lfloor \omega \mathbf{v}_i(t) + c_1 r_1 \left(\mathbf{p}_i^{\text{best}}(t) - \mathbf{x}_i(t)\right) + c_2 r_2 \left(\mathbf{g}_i^{\text{best}}(t) - \mathbf{x}_i(t)\right) + c_3 r_3 \Big\rfloor \tag{59}$$

where f_{th} is a predefined threshold, and r_3 is a random number uniformly distributed in [0, 1]. Here, the fitness function of the ith particle is given by

$$f_i(\cdot) = \beta_1 \lambda_1 p_1 + \beta_2 \lambda_2 p_2 - \beta_3 \Big(\lambda_1 \left(\alpha T_1 \left(\lambda_1, g_{i,1}^{\text{best}}\right) + p_1\right) + \lambda_2 \left(\alpha T_2 \left(\lambda_2, g_{i,2}^{\text{best}}\right) + p_2\right)\Big) \tag{60}$$

and $\overline{f(t)}$ represents the average value of the fitness of the swarm.

Otherwise, we have

$$\mathbf{v}_i(t+1) = \Big\lfloor \omega \mathbf{v}_i(t) + c_1 r_1 \left(\mathbf{p}_i^{\text{best}}(t) - \mathbf{x}_i(t)\right) + c_2 r_2 \left(\mathbf{g}_i^{\text{best}}(t) - \mathbf{x}_i(t)\right) \Big\rfloor. \tag{61}$$

In addition, the inertial weight ω controls a tradeoff between the local and global search capability. In conventional PSO, the inertial weight ω is usually set as a constant. The disadvantage of a constant ω lies in that a large ω leads to a good global search while a small ω results in a good local search [35]. In order to adjust the search capability dynamically, we introduce a linear decreasing inertia weight in which the value of ω is linearly decreased from an initial value $\omega_{\max}$ to a final value $\omega_{\min}$ by

$$\omega(t) = \omega_{\max} - \frac{\omega_{\max} - \omega_{\min}}{T} t \tag{62}$$

where T is the maximum number of iterations. The GPSO algorithm is shown in Algorithm 1.

Algorithm 1 GPSO Algorithm

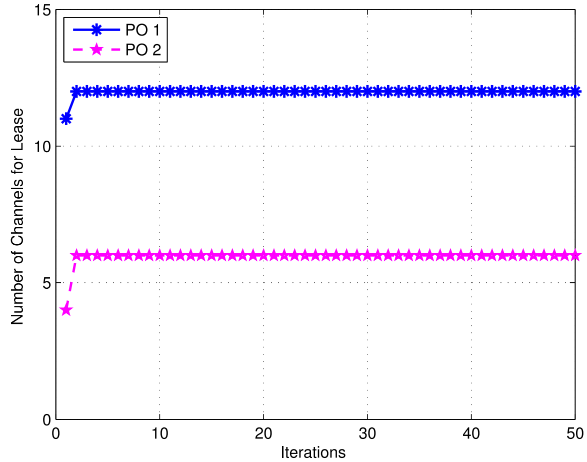
Initialize $\mathbf{s}_i^{(0)} = (s_{i,1}^{(0)}, s_{i,2}^{(0)}, \ldots, s_{i,D}^{(0)})$ and $\mathbf{v}_i^{(0)} = (v_{i,1}^{(0)}, v_{i,2}^{(0)}, \ldots, v_{i,D}^{(0)})$; Set $t = 0$
Compute the fitness function of each particle as (60), and the average value $\overline{f(0)}$
Initialize the best previous position $\mathbf{p}_i^{\text{best}}(0) = \mathbf{s}_i^{(0)}$, and set the best position $\mathbf{g}_i^{\text{best}}(0) = \arg\max_{\mathbf{p}_i^{\text{best}}} f(\mathbf{p}_i^{\text{best}}(0))$
for $t = 1$ to the maximum iteration number **do**
  for all i **do**
    Call (62) to compute the inertial weight
    if $|\overline{f(t)} - f_i(\mathbf{g}_i^{\text{best}}(t))| \le f_{\text{th}}$ **then**
      Compute the velocity as (59)
    end if
    if $|\overline{f(t)} - f_i(\mathbf{g}_i^{\text{best}}(t))| > f_{\text{th}}$ **then**
      Compute the velocity as (61)
    end if
    Update the position as (56)
    Compute the best previous position $\mathbf{p}_i^{\text{best}}$ as (57)
    Compute the global best position $\mathbf{g}_i^{\text{best}}$ as (58)
    $t = t + 1$
  end for
end for
return $\mathbf{s}_i^*$

C. Numerical Study

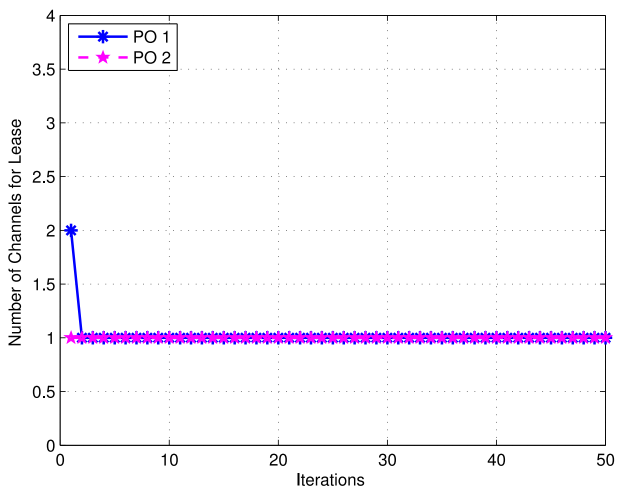
We now utilize numerical results to verify the effectiveness of the proposed GPSO algorithm. Here, we define $s_1^{\max} = 12$, $s_2^{\max} = 6$, $\beta_1 = 0.4$, $\beta_2 = 0.4$, $\beta_3 = 0.2$, and c_1, c_2, and c_3 are all set to 2. Fig. 9(a) shows the evolution of the number of bands to be leased by two POs. It is found that the GPSO algorithm converges very fast. After about four iterations, these two numbers reach the optimal values of 9 and 4, respectively. Thereafter, we change the weights to $\beta_1 = 0.1$, $\beta_1 = 0.1$, $\beta_3 = 0.8$. It is shown in Fig. 9(b) that both POs would lease all the idle bands to SUs. As β_1, β_2, and β_3 keep a tradeoff among the utilities of PO1, PO2, and SUs. With a larger β_3, the minimization of the overall cost of SUs becomes the dominant problem in (51), which requires the POs to rent all their idle bands so as to reduce the queueing delay of SUs.

Fig. 10 compares the performance of the proposed GPSO algorithm with the traditional PSO algorithm. It is observed from the upper and middle figure that the convergence performance of the PSO is not stable. Depending on the settings of the inertial weight ω, the PSO may converge to the global optimum ($\omega = 1$) or a local optimum ($\omega = 3$). While our GPSO algorithm always converges to the global optimum by introducing a linear decreasing ω. Furthermore, to demonstrate the effectiveness of the proposed velocity update formula in (59), we fix ω to 3 for both GPSO and PSO algorithm, and then compare their convergence performance. It is shown in the lower figure that the proposed velocity formula helps to reach the global optimum.

Finally, Fig. 11 compares the average queueing delay and revenue of two POs, where we use the same settings as in Fig. 9(a). Namely, we have $s_1^{\max} = 12$, $s_2^{\max} = 6$, $s_1^* = 9$,



(a)



(b)

Fig. 9. Number of the bands for rent. (a) $\beta_1 = 0.4$, $\beta_2 = 0.4$, and $\beta_3 = 0.2$. (b) $\beta_1 = 0.1$, $\beta_1 = 0.1$, and $\beta_3 = 0.8$.

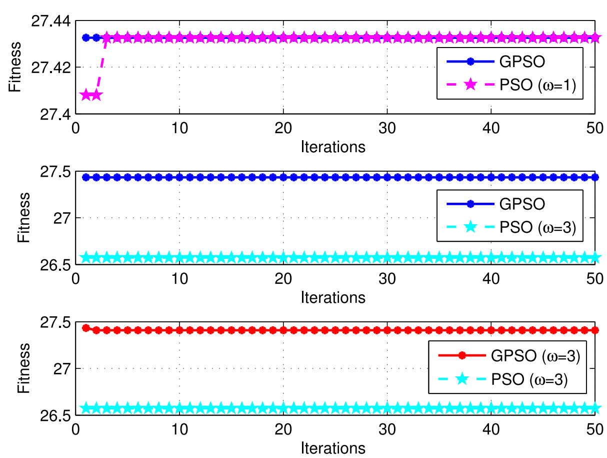
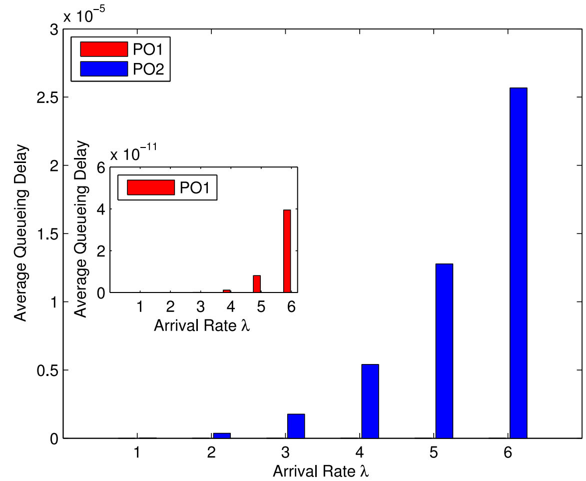
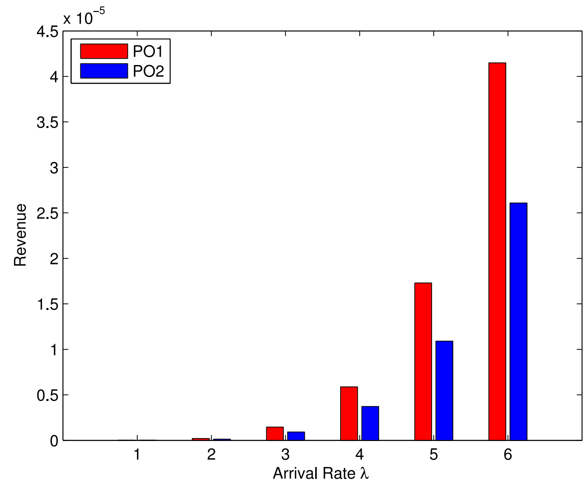


Fig. 10. Comparison of GPSO with traditional PSO.

$s_2^* = 4$. It is observed that the queueing delay provided by PO 1 is much lower than that by PO 2. This is because that PO 1 leases much more bands to SUs, which greatly reduces the average queueing delay of the SUs joining PO 1 band. As



(a)



(b)

Fig. 11. Performance comparison of two POs.

a reward, PO 1 achieves more revenue than PO 2 from leasing channel.

VII. Implementation and Extension

We now discuss the implementation of the proposed pricing strategies. In practice, both the parallel and leader–follower strategy can be implemented in a distributed way. Here, we take the parallel strategy as an example. Assume that there exists an error-free control channel between two POs, via which two POs can exchange the service rate μ_i and the achieved equilibrium arrival rate λ_i^{WE} with each other. First, PO 1 calculates the equilibrium arrival rate λ_1^{WE} and λ_2^{WE} by (23), where PO 2′s service rate should be informed. Thereafter, PO 1 sends back the information of its service rate and the equilibrium arrival rate achieved to PO 2. Finally, both POs independently find out their optimal prices based on (19) and (20). For multiband pricing strategy, its low level optimization procedure can be distributedly implemented by either the parallel or the leader–follower strategy, and its high level optimization procedure can be executed by the GPSO algorithm in a centralized way.

Finally, we discuss the extension of the proposed pricing and decision-making strategies to the competition of multi-PO multiqueue. For single-band access scenario, the parallel strategy can be used straightforwardly. However, the formulations of SUs′ queueing delay would be more complicated. Specifically, assume that SUs, joining queue $j = 1, 2, \ldots, N$ of PO $i = 1, 2, \ldots, M$, arrive with a Poisson process of rate $\lambda_{i,j}$. Let $T_{i,j}$ be the expected queueing delay of SUs joining queue j of PO i, and $T_{i,1\sim j}$ be the expected queueing delay of SUs selecting the first $j = 1, 2, \ldots, N$ queues of PO i, then we have

$$T_{i,1\sim j} = \frac{1}{\left(\mu_i - \sum_{k=1}^{j} \lambda_{i,k}\right)} \tag{63}$$

$$T_{i,j} = \frac{\sum_{k=1}^{j} \lambda_{i,k}}{\lambda_{i,j}} T_{i,1\sim j} - \frac{\sum_{k=1}^{j-1} \left(\lambda_{i,k} T_{i,k}\right)}{\lambda_{i,j}}. \tag{64}$$

Moreover, the two WE principles will become: 1) the overall costs perceived by SUs choosing any PO $i = 1, 2, \ldots, M$ are all equal and 2) the overall costs perceived by SUs choosing any queue $j = 1, 2, \ldots, N$ of PO i are all equal. Note that the leader–follower strategy, only suitable for two players, becomes invalid in multi-PO case.

For multiband multi-PO access scenario, the expected queueing delay of SUs choosing PO $i = 1, 2, \ldots, M$ is still calculated by (48). At this time, the optimization problem becomes

$$\begin{aligned} \max\ & \sum_{i=1}^{M} \beta_i \lambda_i p_i - \beta_{i+M} \left(\sum_{i=1}^{M} \lambda_i (\alpha T_i(\lambda_i, s_i) + p_i) \right) \\ & s_i \in \left\{1, 2, \ldots, s_i^{\max}\right\},\ i = 1, 2, \ldots, M \\ & \sum_{i=1}^{M} \lambda_i = \lambda \\ & \lambda_i < \mu_i,\ i = 1, 2, \ldots, M. \end{aligned} \tag{65}$$

By using the decomposition approach, the low level optimization procedure can be solved by the parallel pricing strategy, and the high level procedure can be solved by the GPSO algorithm.

VIII. Conclusion

In this paper, we studied priced-based spectrum access control in CR networks consisting of POs, PUs, and SUs. Each PO serves two queues for heterogeneous SU demand. We considered three types of duopoly markets with three kinds of queueing models. In single-band markets, two pricing strategies are developed for maximizing the revenue of POs. The existence of a unique WE point is proved for the incoming traffic partition of SUs, at which the average cost of SUs is minimum. In multiband market, POs need to determine the number of bands to be leased and the corresponding admission price. The optimization problem is then formulated as an MILP problem. In order to find the solution in polynomial time, a decomposition approach is proposed, by which the MILP problem is decomposed into a pricing subproblem and an IP subproblem for determining the leased band numbers. A GPSO algorithm is presented for solving the IP subproblem. Finally, the extension of the proposed pricing and decision-making strategies to multi-PO and multiqueue scenarios are discussed. Future work would be expanded to more complicated scenarios, for example, we may consider more complicated traffic patterns and queueing models for PUs and SUs, and take sensing errors of SUs into account.

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