## Optimal Spatio-Temporal Projections with Holo-Kronecker Compressive Sensing of Video Acquisition

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The emergence of compressive sensing(CS) offers a sub-Nyquist sampling system for the growing size of multimedia data. However, multidimensional signals require more consideration on the tradeoff between enjoying extra structured sparsity and implementing low-complexity samplings. Recently, Duarte et al. [1] introduce an innovative CS system that employs the Kronecker product to meet the space-time structure of video. In this paper, we proposes a highly compressed video sampling scheme that optimally utilizes the redundancy spanning all the dimensions of the signal. Considering that we further compress the redundant measurements of the existing Kronecker Compressive Sensing(KCS), the proposed scheme will significantly improve the sampling efficiency.

The contribution of our work is two fold. First, a holo-compressive sampling spanning both temporal and spatial dimensions is proposed to overcome the drawback of KCS, and named HKCS. This new scheme keeps on employing Kronecker product to design the sensing matrix. However, it is worth emphasizing that the original identity matrix, a factor of the conventional Kronecker product sensing matrix, is replaced by an ill-posed matrix which enables compressive samplings along the temporal dimension. The advantages of HKCS include: samplings of video along the temporal and spatial dimensions are simultaneously compressed and the necessary measurements for exact recovery are significantly reduced; moreover, the Kronecker product sensing matrix retains the block structure, which ensures a feasible distributed sampling scheme for practical application. We prove that, with the same sampling rate, matrices of HKCS have relatively smaller mutual coherence. It can be written as

$$\mu(\hat{\Phi}_{Proposed}, \hat{\Psi}) \le \mu(\hat{\Phi}_{KCS}, \hat{\Psi}). \tag{1}$$

 $\hat{\Psi}$  and  $\hat{\Phi}$  represent the corresponding sparsifying and sensing matrices. This conclusion theoretically demonstrates the superiority of our method. The second part of our work is that we adopt an optimization for the sensing matrix, thus obtain an optimal sensing matrix that minimizes the mutual coherence of the projection matrix. The overall mutual coherence is decomposed as

$$\mu((\Phi_1 \otimes \ldots \otimes \Phi_D) \cdot (\Psi_1 \otimes \ldots \otimes \Psi_D)) = \mu(\Theta_1) \times \ldots \times \mu(\Theta_D). \tag{2}$$

Here  $\Theta_i = \Phi_i \Psi_i$ . With the divisibility of mutual coherence, we can obtain the optimal synthetic sensing matrix by taking Kronecker products of individual optimal sensing matrix in each dimension. It also indicates that the optimization process is dividable, which preserves the block feature of Kronecker product matrix and enables fast low-scale matrix computation.

Experiments on standard video datasets show that the gain is achieved by the HKCS over KCS about 1-3 dB in terms of PSNR. We also discuss the allocation of the sampling rates that provides best recovery performance. Furthermore, it is found that the optimization for the sensing matrix brings not only more exact reconstruction but also faster convergence to the solution of the convex optimization.

## Reference

[1] M.F.Duarte, R.G.Baraniuk, "Kronecker Compressive Sensing" to appear in IEEE Trans. Image Processing.

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