上海交通大學
SHANGHAI JIAO TONG UNIVERSITY

Institute of Media,
Information, and Network

Chapter 1 Signals and Systems

熊红凯
特聘教授
http://min.sjtu.edu.cn

电子工程系
上海交通大学

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About Me

- Hongkai Xiong, distinguished professor
  - Office: 1-309, No.1, SEIEE Bld.
  - Email: xionghongkai@sjtu.edu.cn
  - Web-page: http://min.sjtu.edu.cn

About TA

- Xing Gao, Phd candidate
- Email: william-g@sjtu.edu.cn
- Yuhui Xu, Phd candidate
- Email: yuhuixu1993@126.com
- Office: R304, No.1, SEIEE Bld. (both)

About The Class

- Requirements and Grading:

  - Homework & Project + Mid-term Test: 50%

  - Final Exam : 50%

About The Class

- Text book and reference:
  - Signals & Systems (Second Edition)
    by Alan V. Oppenheim, 电子工业出版社

References

《信号与系统》刘树棠，西安交通大学出版社

《复变函数》严镇军，中国科学技术大学出版社

《信号与系统》（上、下）郑君理，高等教育出版社

Topic

- 1.0 INTRODUCTION
- 1.1 CONTINUOUS-TIME AND DISCRETE-TIME SIGNALS
- 1.2 TRASFORMATION OF INDEPENDENT VARIABLE
- 1.3 EXPONENTIAL AND SINUSOIDAL SIGNALS
- 1.4 THE UNIT IMPULSE AND UNIT STEP FUNCTIONS
- 1.5 Definitions and Representations of Systems
- 1.6 BASIC SYSTEM PROPERTIES

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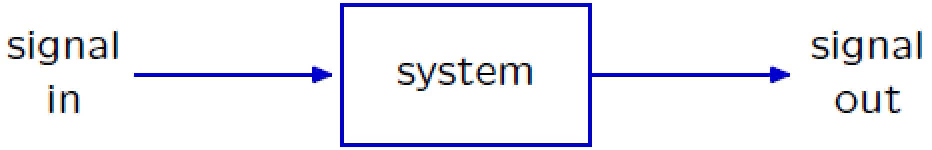
Introduction

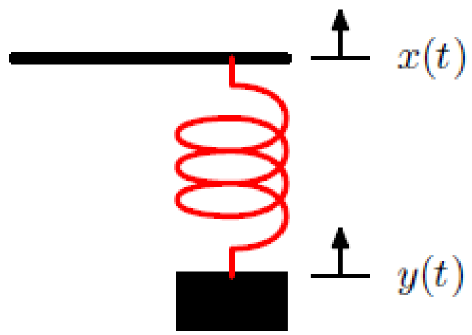
- Signals
  - Definitions, representations and classifications
  - Fundamental signal transformations
  - Typical signal examples
- Systems
  - Concepts, representations, and classifications
  - Basic properties of systems

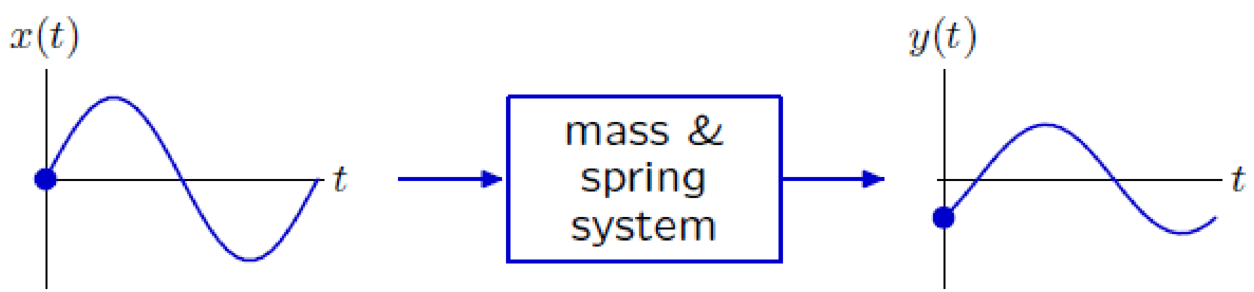
The Signals and Systems Abstraction

- Describe a system (physical, mathematical, or computational) by the way it transforms an input signal into an output signal.

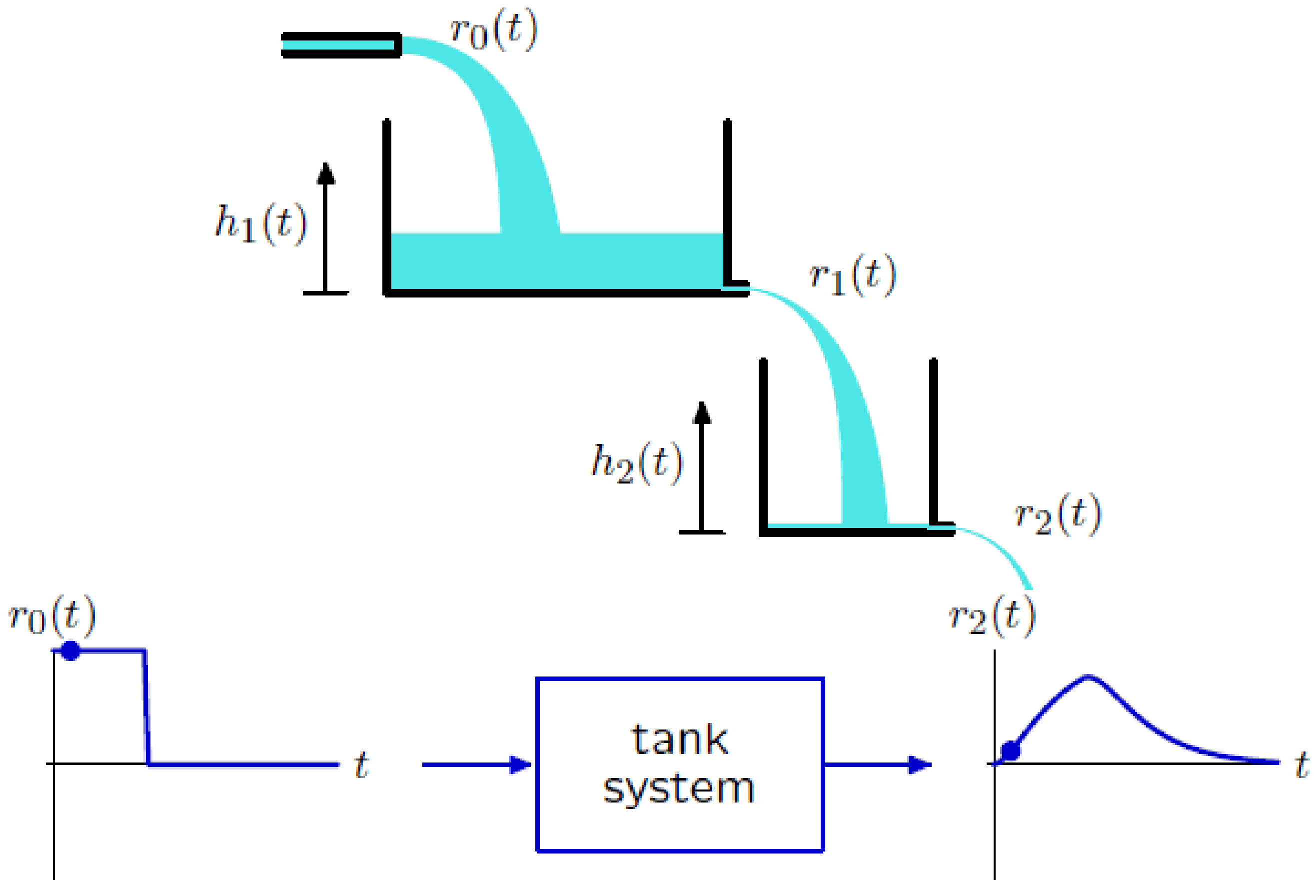


Example: Mass and Spring

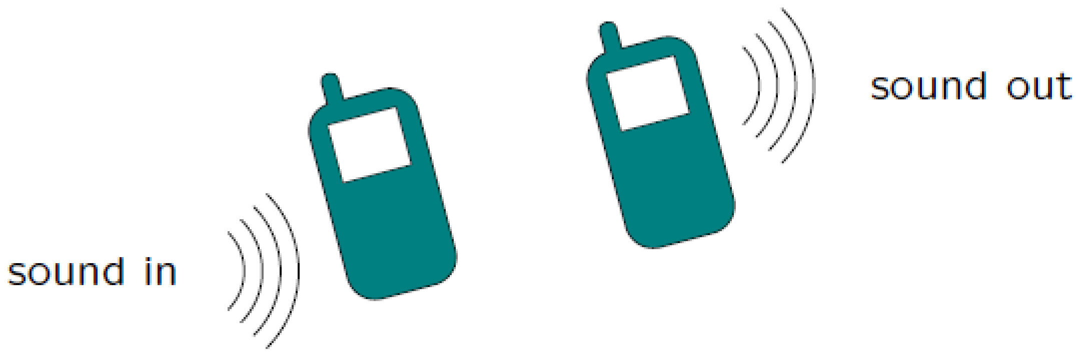


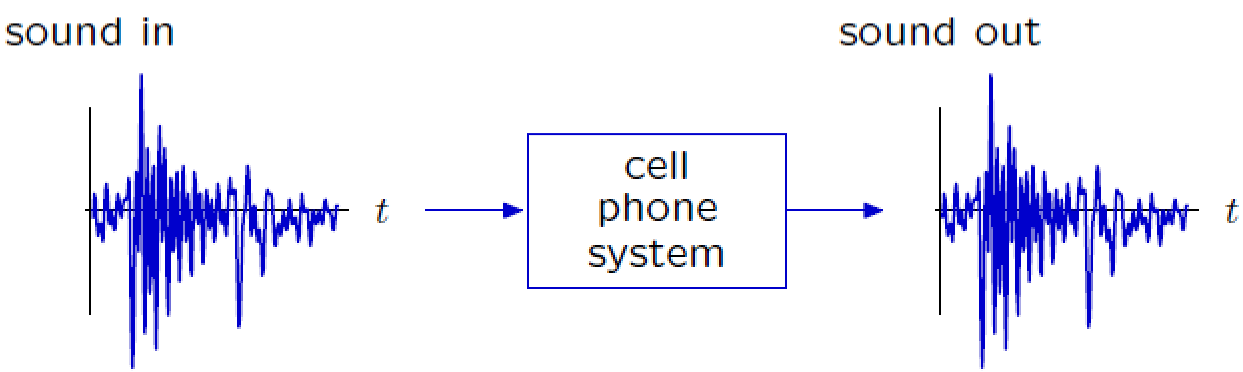


Example: Tanks



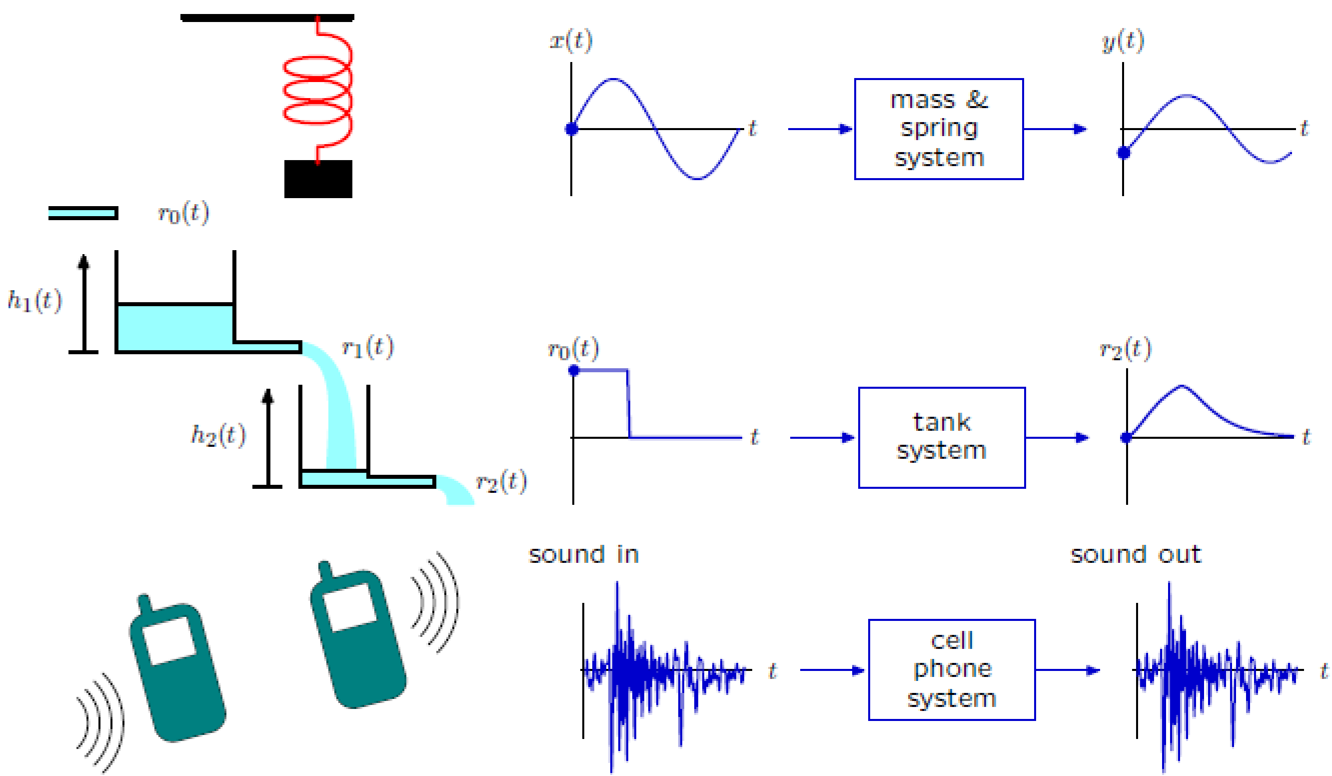
Example: Cell Phone System





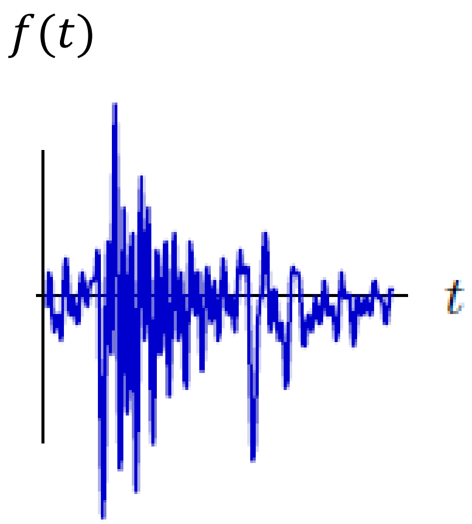
Signals and Systems: Widely Applicable

- The Signals and Systems approach has broad application: electrical, mechanical, optical, acoustic, biological, financial, ...



Check Yourself

- Computer generated music $f(t)$



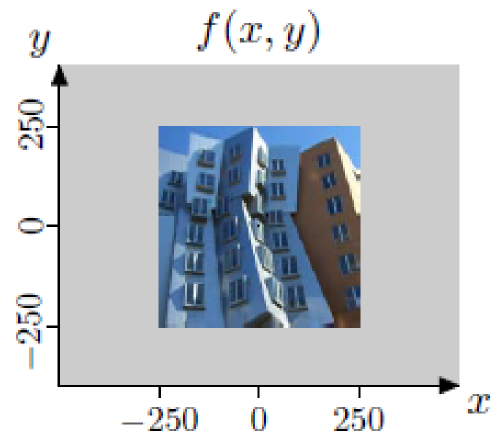
Little Test

- Listen to the following three manipulated signals:

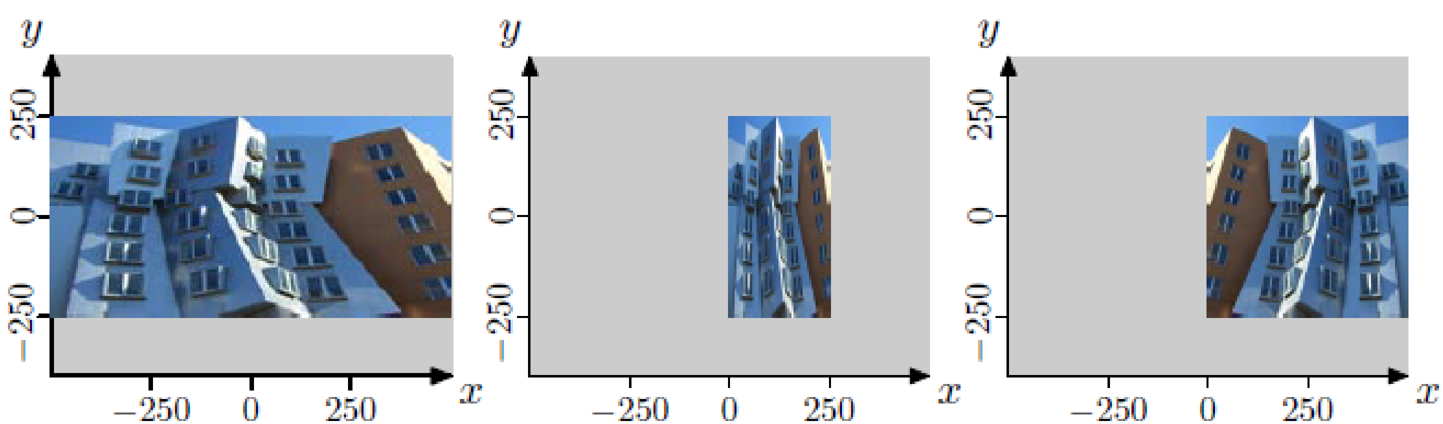
$f_1(t)$ $f_2(t)$ $f_3(t)$, try to find the correct answer

$f_1(t)$	$-f(t)$
$f_2(t)$	$0.5f(t)$
$f_3(t)$	$f(2t)$

Check Yourself

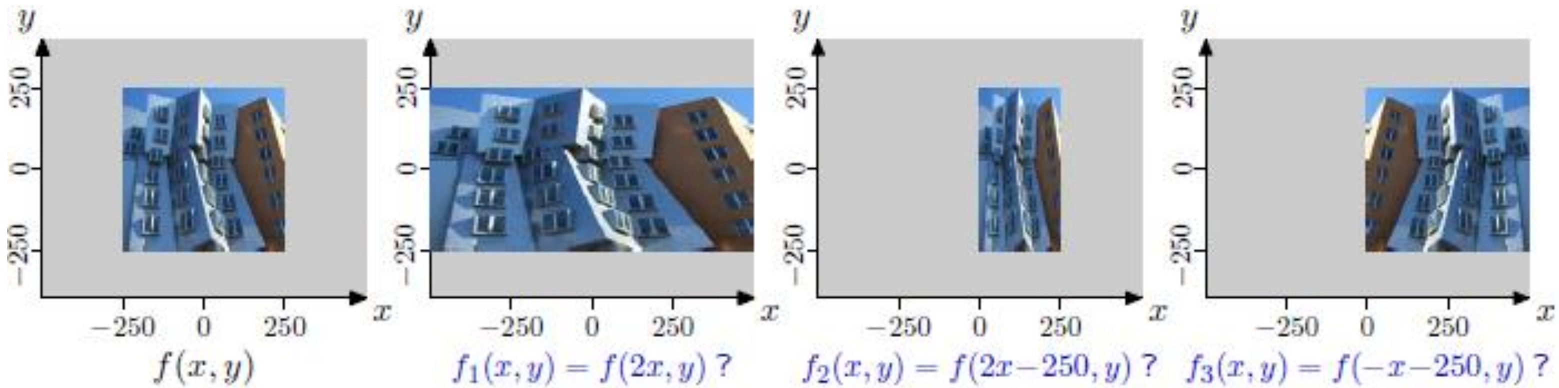


How many images match the expressions beneath them?



$f_1(x,y) = f(2x,y)$? $f_2(x,y) = f(2x-250,y)$? $f_3(x,y) = f(-x-250,y)$?

Check Yourself



$$x = 0 \quad \rightarrow f_1(0, y) = f(0, y) \quad \checkmark$$
$$x = 250 \quad \rightarrow f_1(250, y) = f(500, y) \quad \times$$

$$x = 0 \quad \rightarrow f_2(0, y) = f(-250, y) \quad \checkmark$$
$$x = 250 \quad \rightarrow f_2(250, y) = f(250, y) \quad \checkmark$$

$$x = 0 \quad \rightarrow f_3(0, y) = f(-250, y) \quad \times$$
$$x = 250 \quad \rightarrow f_3(250, y) = f(-500, y) \quad \times$$

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1.1.1 Mathematical Representation of Signals

- Signals are represented mathematically as functions of one or more independent variables

- Described by mathematical expression and waveform

(In this book, we focus our attention on signals involving a single independent variable as time)

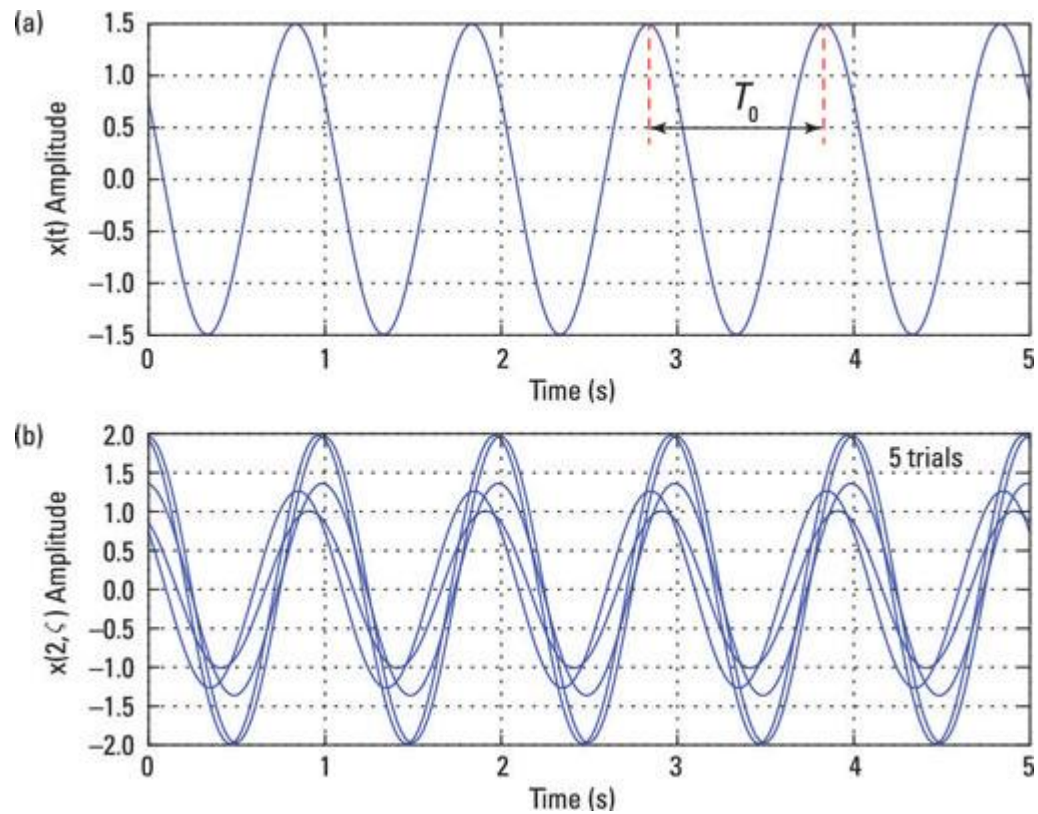
1.1.2 Classification of Signals

- Deterministic signal and Random Signal
- Continuous signal and Discrete Signal
- Energy Signal and Power Signal
- Periodic Signal and Non-periodic Signal
- Odd Signal and Even Signal
- Real Signal and Complex Signal
- ………………….

1.1.2.1 Deterministic signal and Random Signal

- Deterministic signal
  - Can be described by exact Mathematic expression
  - Given t and get Deterministic result

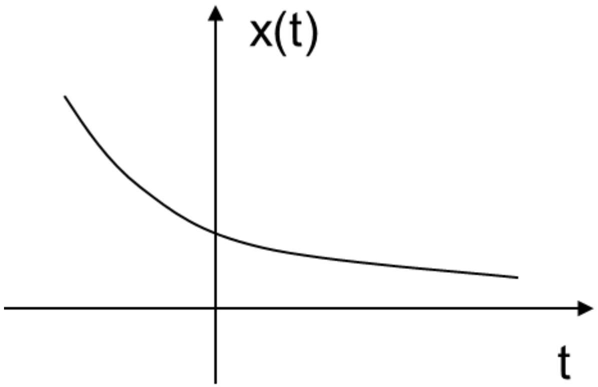
- Random Signal
  - Can not be described by exact Mathematic expression
  - Given t and get random result



1.1.2.2 Continuous-Time (CT) and Discrete-Time (DT) Signals:

- Continuous-Time (CT) Signals: x(t)
  - Independent variable (t) is continuous
  - The signal is defined for a continuum of values of the independent variable (t)

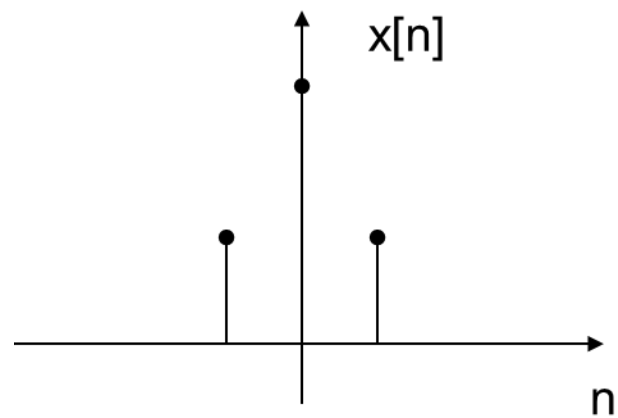
$$example: \quad x(t) = 2e^{-t}$$



1.1.2.2 Continuous-Time (CT) and Discrete-Time (DT) Signals:

- Discrete-Time (DT) Signals/Sequences: x[n]
    - Independent variable (n) takes on only a discrete set of values, in this course, a set of integer values only
    - Signal is defined only at discrete times

$$example: \quad x[n] = \begin{cases} 2, & n=-1 \\ 4, & n=0 \\ 2, & n=1 \\ 0, & others \end{cases}$$



1.1.2.3 Time-Limited and Power-Limited Signals

Power and energy in a physical system

- Instantaneous power $$P(t) = v(t)i(t) = \frac{1}{R}\left|v(t)\right|^2$$
- Total energy over time interval $[t_1, t_2]$ $$\int_{t_1}^{t_2} p(t)dt = \frac{1}{R}\int_{t_1}^{t_2} \left|v(t)\right|^2 dt$$
- Average power over time interval $[t_1, t_2]$ $$\frac{1}{t_2 - t_1}\int_{t_1}^{t_2} p(t)dt = \frac{1}{t_2 - t_1}\frac{1}{R}\int_{t_1}^{t_2} \left|v(t)\right|^2 dt$$

1.1.2.3 Time-Limited and Power-Limited Signals

Power and energy definitions in the course

- Total Energy

$$E \overset{\Delta}{=} \int_{t_1}^{t_2} |x(t)|^2 \, dt \qquad\qquad E \overset{\Delta}{=} \sum_{n=n_1}^{n_2} |x[n]|^2$$

- Average Power

$$P \overset{\Delta}{=} \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} |x(t)|^2 \, dt \qquad\qquad P \overset{\Delta}{=} \frac{1}{n_2 - n_1 + 1} \sum_{n=n_1}^{n_2} |x[n]|^2$$

1.1.2.3 Time-Limited and Power-Limited Signals

Power and energy definitions over an infinite interval

- Total Energy

$$E_\infty = \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2 dt \qquad E_\infty = \lim_{N\to\infty} \sum_{n=-N}^{N} |x[n]|^2$$

- Average Power

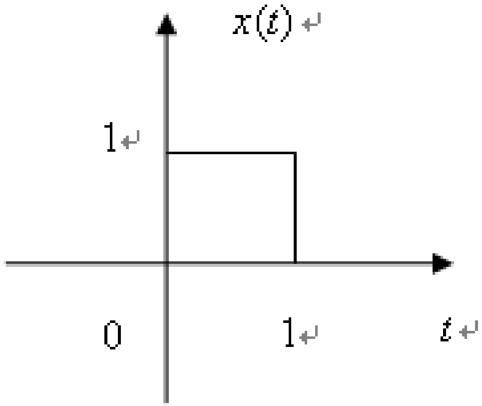
$$P_\infty = \frac{1}{2T} \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2 dt \qquad P_\infty = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2$$

1.1.2.3 Time-Limited and Power-Limited Signals

- Finite-Energy Signal

$$E_\infty < \infty \qquad P_\infty = 0$$

example:



- Finite-Average Power Signal

$$P_\infty < \infty \qquad E_\infty = \infty$$

example: $x[n]=4$

1.1.2.4 Periodic and Non-Periodic Signals

For continuous-time signals

- Definition:

  If $x(t) = x(t+T)$ for all values of t, $x(t)$ is periodic Then $x(t) = x(t+mT)$ for all t and any integral m

  - Fundamental Period: the smallest positive value of satisfying $x(t) = x(t+T)$ for all t

If the signal is constant,
the fundamental period ?

1.1.2.4 Periodic and Aperiodic Signals

For discrete-time signals

- Definition:

  If $x[n] = x[n+N]$ for all values of n, $x[n]$ is periodic Then $x[n] = x[n+mN]$ for all n and any integral m

  - Fundamental Period: the smallest positive value of satisfying $x[n] = x[n+N]$ for all n

If the signal is constant,
the fundamental period ?

1.1.2.5 Even and Odd Signals

- Definition:

  $x(t)$ or $x[n]$ is even if it is identical to its time-reversed counterpart

$$x(t) = x(-t) \qquad\qquad x[n] = x[-n]$$

  Similarly $x(t)$ or $x[n]$ is odd if

$$x(t) = -x(-t) \qquad\qquad x[n] = -x[-n]$$

For odd signal $x(t)$, can one determine $x(0)$?

1.1.2.5 Even and Odd Signals

- Even-odd decomposition of a signal

$$x(t) = \underbrace{E_v\{x(t)\}}_{\text{Even part}} + \underbrace{O_d\{x(t)\}}_{\text{Odd part}}$$

$$E_V\{x(t)\} = \frac{1}{2}[x(t) + x(-t)] \qquad O_d\{x(t)\} = \frac{1}{2}[x(t) - x(-t)]$$

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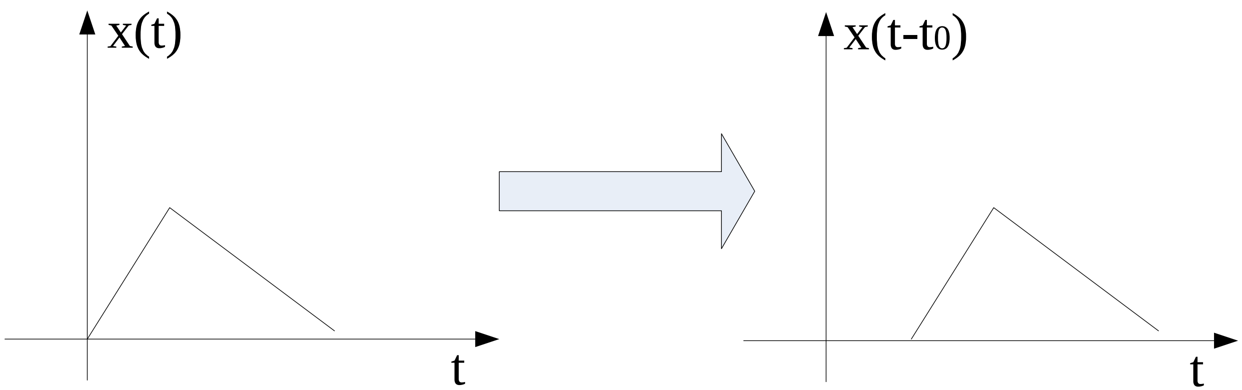
1.2.1 Time Shift

$$x(t) \to x(t - t_0) \qquad\qquad x[n] \to x[n - n_0]$$



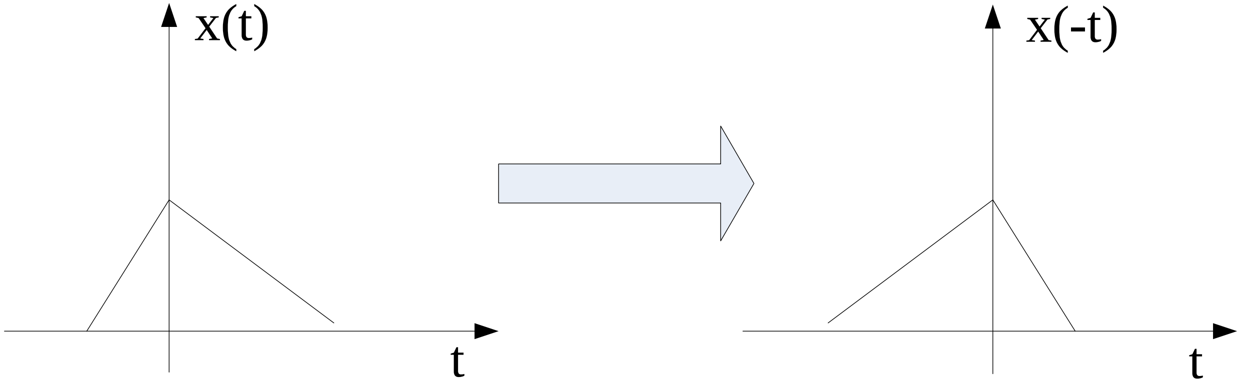
e.g.: Radar, Sonar, Radio propagations

Notes: Each point in x(t)/x[n] occurs at a later/early time in $x(t-t_0)/x[n-n_0]$, when t_0/n_0 is positive/negative, i.e.

- $x(t-t_0)/x[n-n_0]$ is the delayed version of x(t)/x[n], for $t_0/n_0 > 0$
- $x(t-t_0)/x[n-n_0]$ is the advanced version of x(t)/x[n], for $t_0/n_0 < 0$

1.2.2 Time Reversal

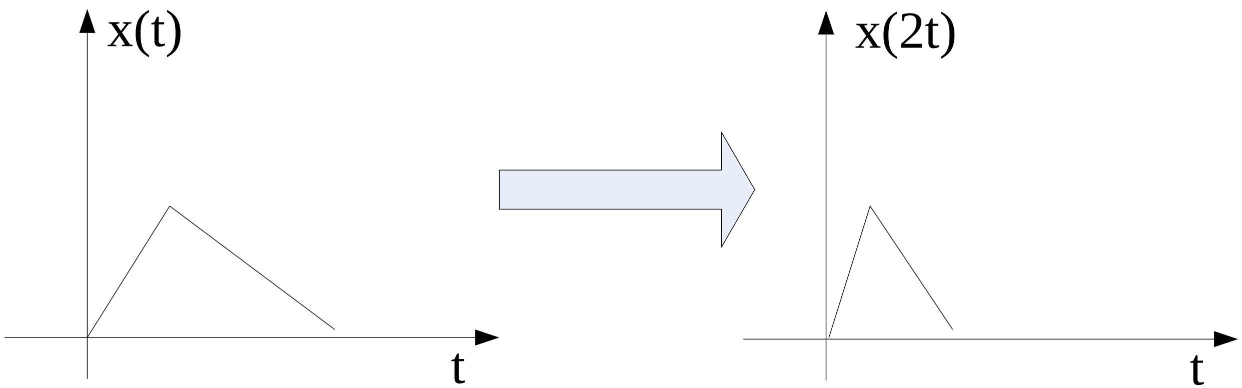
$$x(t) \to x(-t) \qquad x[n] \to x[-n]$$



e.g.: tape recording played backward

1.2.3 Time Scaling

$$x(t) \rightarrow x(\alpha t) \qquad x[n] \rightarrow x[\alpha n]$$



E.g. tape recording played:

- fast forward $\alpha > 1$
- slow forward $0 < \alpha < 1$
- slow backward $-1 < \alpha < 0$
- fast backward $\alpha < -1$

Notes：

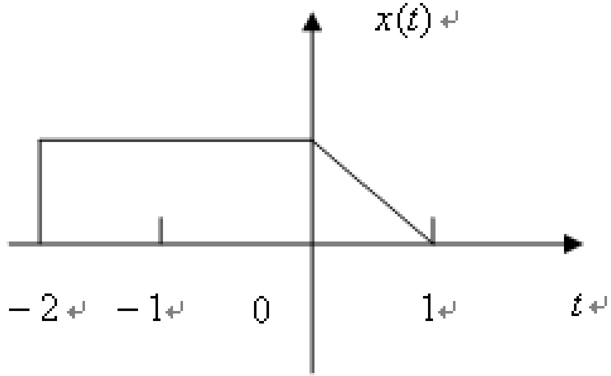
$|\alpha| > 1$ —Compression

$|\alpha| < 1$ —Extension

1.2.4 A General Transform of the Independent Variable

$$x(t) \rightarrow x(\alpha t + \beta) \qquad\qquad x[n] \rightarrow x[\alpha n + \beta]$$

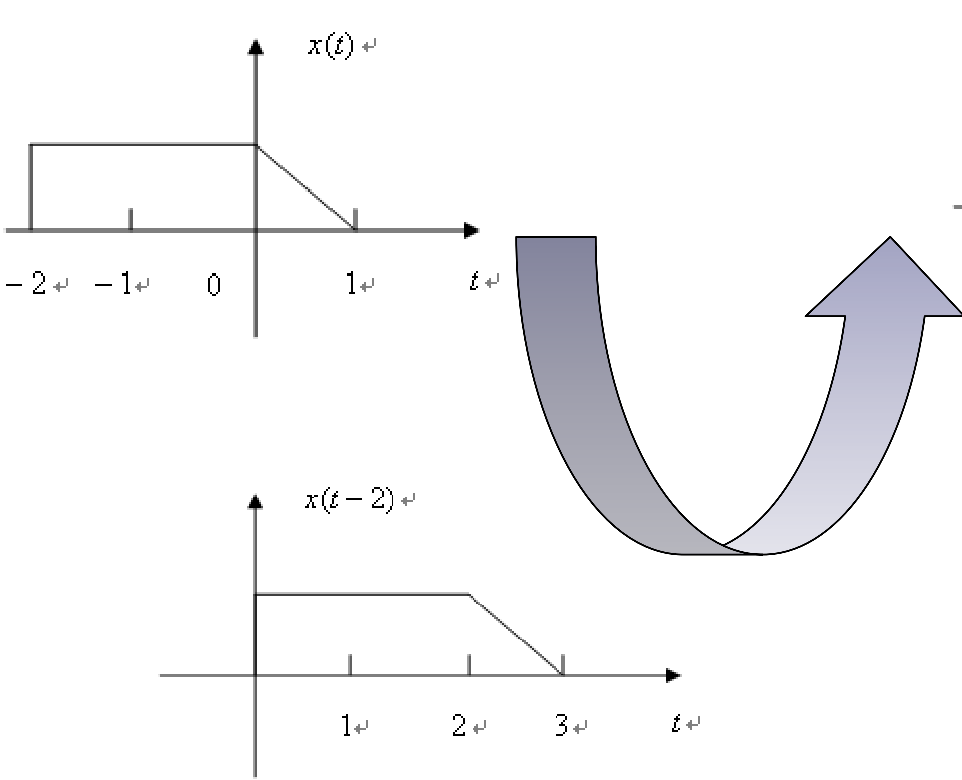
example : $x(t) \rightarrow x(-3t - 2)$

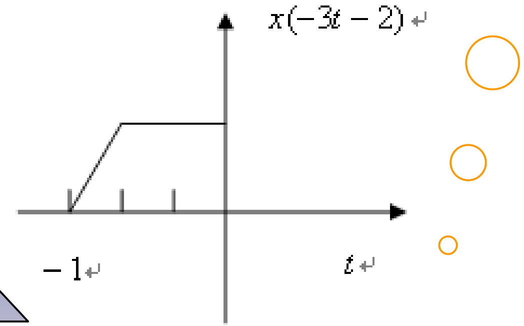


Rule:

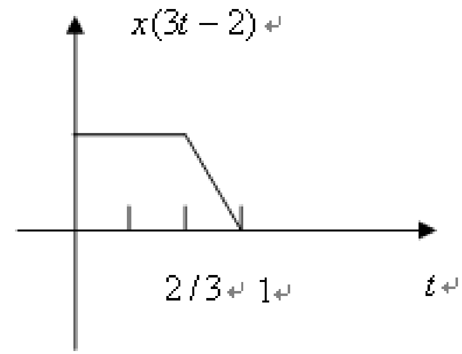
1. **time shift first**
2. **then reflection(time reversal) and time scaling**

Question: what happens if shifting after scaling/reflection

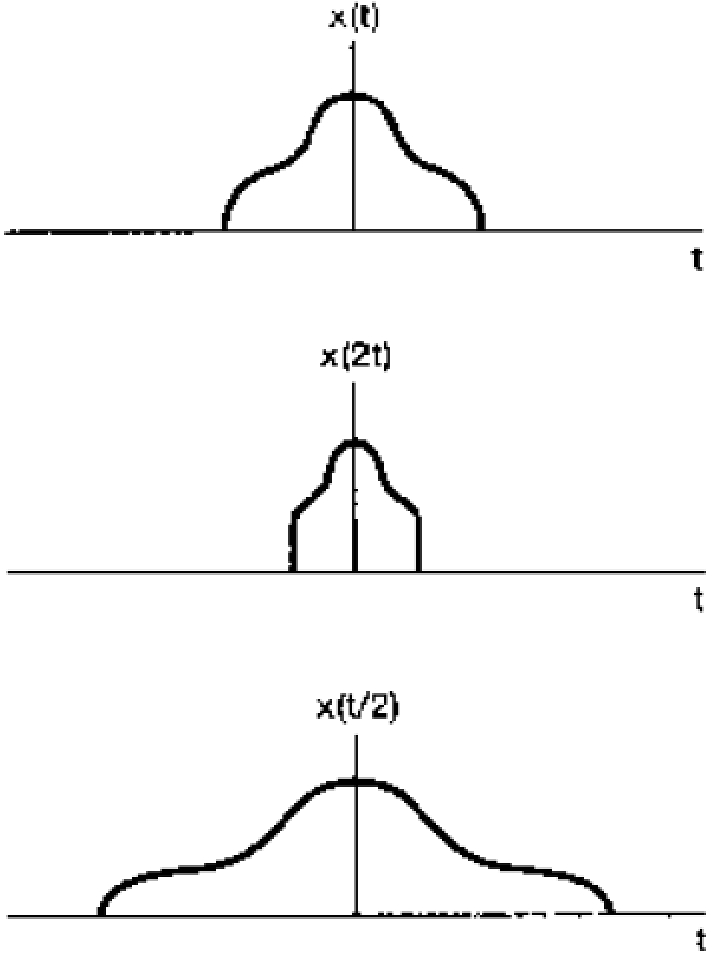


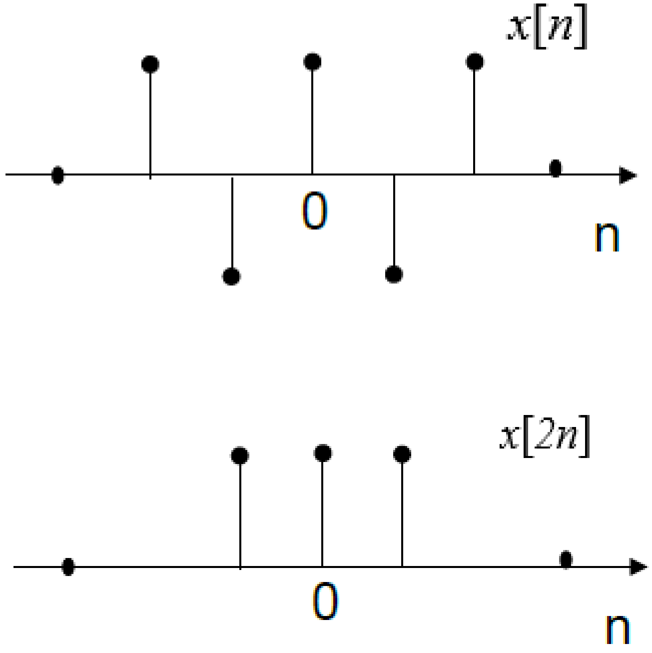


x(-3(t+2/3))



Example: x(t)、x(2t)、x(t/2) *Example: x[n]、x[2n]*





Topic

1.3.1 Continuous-Time Complex Exponentials Signals and Sinusoidal Signals

$$x(t) = C \cdot e^{\alpha t}$$

Where C and α are complex numbers

- Real Exponential Signals: when C and α are real numbers, e.g. $x(t) = e^{2t}$
  - growing exponential, when $\alpha > 0$
  - decaying exponential, when $\alpha < 0$
  - constant $\alpha = 0$

- Periodic Complex Exponential and Sinusoidal Signals: when C is real, α is purely imaginary, e.g.
then the **fundamental period** $T_0 = 2\pi/\omega_0$ [s] , angular frequency ω_0[rad/s], and frequency $f_0 = \frac{\omega_0}{2\pi} = 1/T_0$[Hz]
Unless noted otherwise, in this course, we always call ωo frequency

Euler's Relation

$$e^{j\omega_0 t} = \cos \omega_0 t + j \sin \omega_0 t$$

$$\cos \omega_0 t = \frac{1}{2}(e^{j\omega_0 t} + e^{-j\omega_0 t}) \quad \sin \omega_0 t = \frac{1}{2j}(e^{j\omega_0 t} - e^{-j\omega_0 t})$$

- Important periodicity property :
  - 1) the larger the magnitude of ω_0 , the higher the oscillation in the signal
  - 2) the signal $x(t)$ is periodic for any value of ω_0

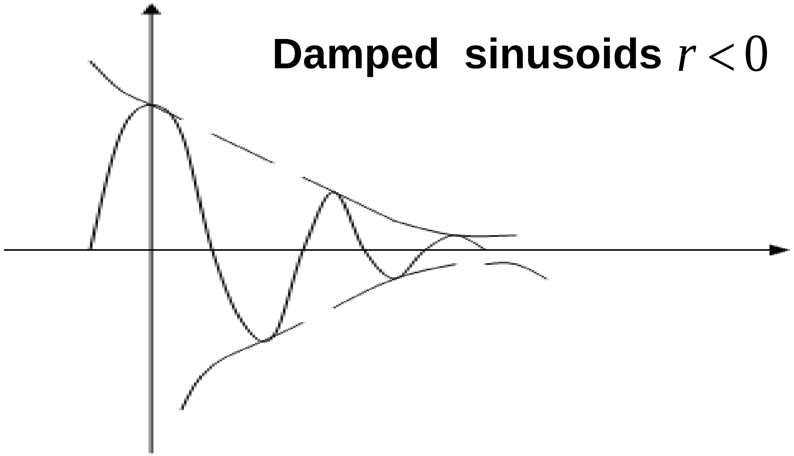
- A general representation, when C and α are complex numbers, denoted as $\mathrm{C} = |\mathrm{C}| e^{j\theta}, \quad \alpha = r + j\omega_0$, then

$$x(t) = |c| \cdot e^{j\theta} \cdot e^{(r + j\omega_0)t} = |c| \cdot e^{rt} \cdot e^{j(\omega_0 t + \theta)}$$

$|c| \cdot e^{rt}$ is the envelop of the waveform

ω_0 is the oscillation frequency

Example of real part of $x(t)$



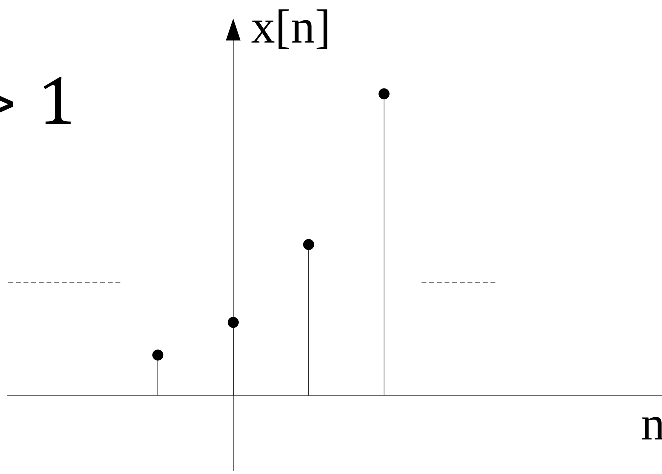
1.3.2 Discrete-Time Complex Exponentials Signals and Sinusoidal Signals

$$x[n] = C \cdot \alpha^n$$

Where C and α are complex numbers

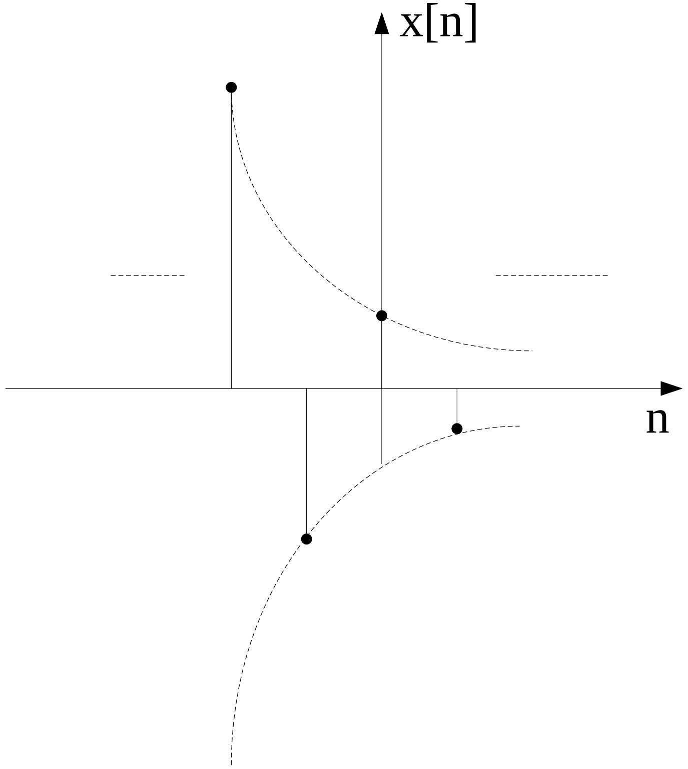
- Real Exponential Signals: when C and α are real numbers
  - e.g. growing function, when $|\alpha| > 1$

$$x[n] = 2^n$$



- decaying function, when $0 < |\alpha| < 1$

$$x[n] = (-1/2)^n$$



- constant, when $|\alpha| = 1$

- alternates in set $\{-C, C\}$, when $|\alpha| = -1$

- Complex Exponential and Sinusoidal Signals: when C is real, α is a point on the unit circle, e.g.

$$x[n] = e^{j\omega_0 n} \quad \text{or} \quad x[n] = A\cos(\omega_0 n + \phi), A\sin(\omega_0 n + \phi)$$

Its periodicity property? Similar to that of continuous-time signals?

- A general representation, when C , α are complex numbers, denoted as $\mathrm{C} = |\mathrm{C}|e^{j\theta}, \quad \alpha = re^{j\omega_0}$,then

$$x[n] = |c| \cdot e^{j\theta} \cdot r^n e^{j\omega_0 \mathrm{n}} = |c| \cdot r^{\mathrm{n}} \cdot e^{j(\omega_0 \mathrm{n} + \theta)}$$

- $|c| \cdot r^n$ is the envelop of the waveform

- Periodicity Property of Discrete-time Complex Exponentials $x[n] = e^{j\omega_0 n}$
  - a) recall the definition of the periodic discrete-time signal $x[n] = x[n+N]$ for all n
  - b)if it is periodic, there exists a positive integer N, which satisfies $e^{j\omega_0 n} = e^{j\omega_0 (n+N)} = e^{j\omega_0 n} e^{j\omega_0 N}$ so, it requires $e^{j\omega_0 N} = 1$, i.e. $\omega_0 N = 2\pi m$
  - If there exists an integer satisfying that $2\pi m/\omega_0$ is an integer, i.e. $2\pi/\omega_0$ is rational number , $x[n]$is periodic with fundamental period of $\mathrm{N} = 2\pi m/\omega_0$, where N, m are integers without any factors in common.

    otherwise, $x[n]$ is aperiodic.

Different from that of continuous exponentials

- Another difference from that of CT exponentials

since $e^{j\omega_0 n} = e^{j(\omega_0 + 2\pi m)n}$ for any integer m
the signal is fully defined within a frequency interval of length 2π : $((2m-1)\pi, \quad (2m+1)\pi]$, for any integer m

Distinctive signals for different ω_0 within any 2π region, i.e.
$((2m-1)\pi, \quad (2m+1)\pi]$ for any integer m

Without loss of generalization, for $\omega_0 \in (-\pi, \quad \pi]$, the rate of oscillation in the signal $e^{j\omega_0 n}$ increases with $|\omega_0|$ increases from 0 to π

Important for discrete-time filter design!

- Comparison of Periodic Properties of CT and DT Complex Exponentials/ Sinusoids

$x(t) = e^{j\omega_0 t}$	$x[n] = e^{j\omega_0 n}$
Distinct signals for distinct value of ω_0	Identical signals for values of ω_0 separated by multiples of 2π
Periodic for any choice of ω_0	Periodic only of $\omega_0 = 2\pi m / N$ for some integers $N > 0$ and m
Fundamental angular frequency ω_0	Fundamental angular frequency ω_0 / m, if m and N do not have any factors in common
Fundamental period $\dfrac{2\pi}{\omega_0}$	Fundamental period $m\left(\dfrac{2\pi}{\omega_0}\right)$

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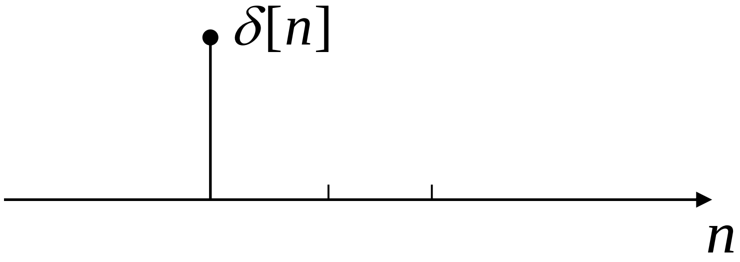
1.4.1 Discrete-Time Unit Impulse and Unit Step Sequences

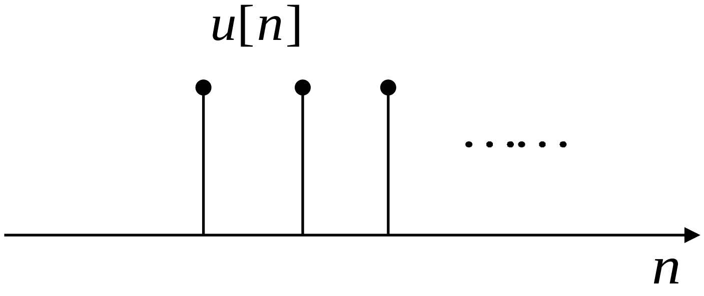
- Unit Impulse Sequence

$$\delta[n] = \begin{cases} 0 & n \neq 0 \\ 1 & n = 0 \end{cases}$$



- Unit Step Sequence

$$u[n] = \begin{cases} 0 & n < 0 \\ 1 & n \geq 0 \end{cases}$$



- Relationship

$$\delta[n] = u[n] - u[n-1]$$ —1st difference

$$u[n] = \sum_{m=-\infty}^{n} \delta[m] \ / \ u[n] = \sum_{k=0}^{\infty} \delta[n-k]$$ —running sum

- Sampling Property

$$x[n]\cdot\delta[n] = x[0]\cdot\delta[n]$$

$$x[n]\cdot\delta[n-n_0] = x[n_0]\cdot\delta[n-n_0]$$

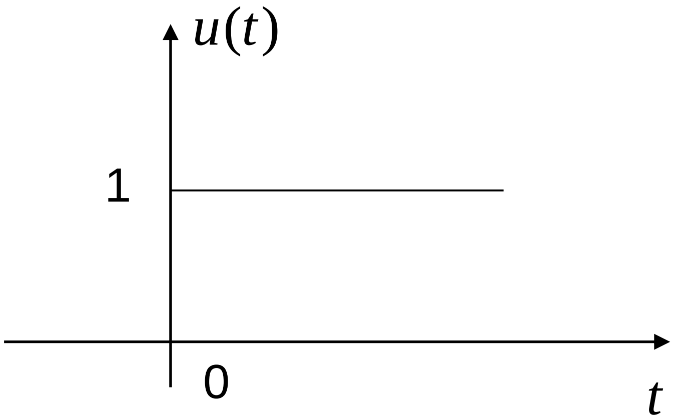
- Signal representation by means of a series of delayed unit samples

$$x[n] = \sum_{k} x[k]\cdot\delta[n-k]$$

1.4.2 Continuous-Time Unit Step and Unit Impulse Functions

- Unit Step Function

$$u(t) = \begin{cases} 0 & t < 0 \\ 1 & t > 0 \end{cases}$$



Notes: $u(t)$ is undefined at $t = 0$

Can we find counterpart of the unit impulse function in CT domain as that in DT domain ?

$$\delta[n] = u[n] - u[n-1]$$ —1st difference

$$u[n] = \sum_{m=-\infty}^{n} \delta[m] \;/\; u[n] = \sum_{k=0}^{\infty} \delta[n-k]$$ —running sum

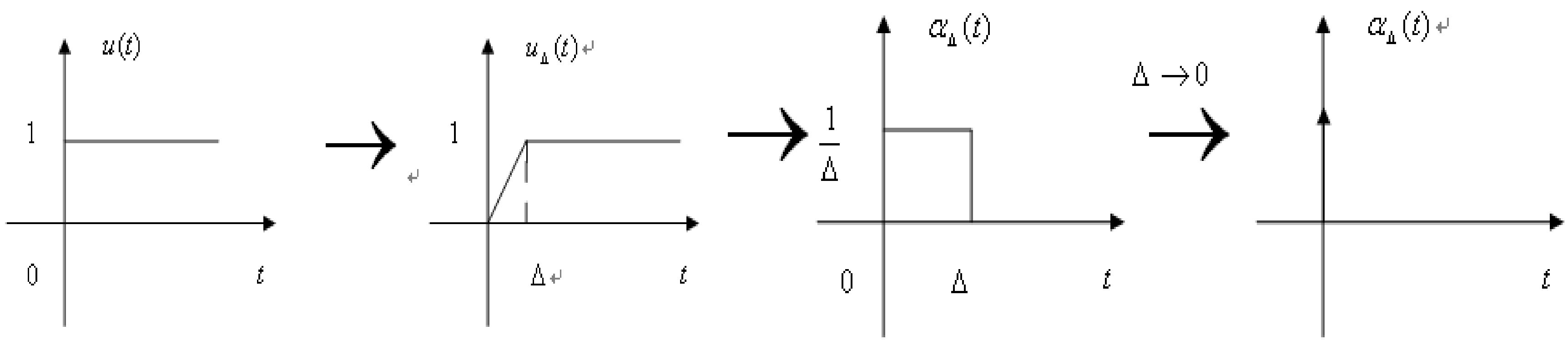
Does it exist $\delta(t)$ satisfying the following relationship

$$\delta(t) = \frac{du(t)}{dt}$$ —1st derivative

$$u(t) = \int_{-\infty}^{t} \delta(\tau)\, d\tau$$ —running sum

?

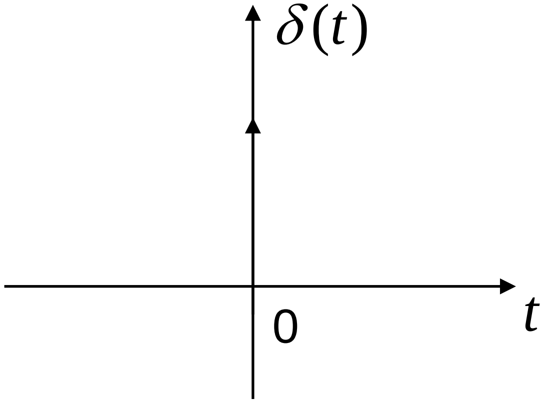
- Unit Impulse Function
  - Since $u(t)$ is undefined at $t = 0$, formally it is not differentiable, then define an approximation to the unit step $u_\Delta(t)$,which rises from 0 to 1 in a very short interval Δ
  - So $\delta_\Delta(t) = \dfrac{d\left(u_\Delta(t)\right)}{dt}$
  - And $\delta(t) = \lim\limits_{\Delta \to 0} \delta_\Delta(t)$



Notes: the amplitude of the signal $\delta(t)$ at $t = 0$ is infinite, but with unit integral from $-\infty$ to ∞ , i.e. from 0^- to 0^+

- Unit Impulse Function
  - Dirac Definition

$$\begin{cases} \int_{-\infty}^{\infty} \delta(t)dt = 1 \\ \delta(t) = 0 \quad t \neq 0 \end{cases}$$



Notes: the amplitude of the signal $\delta(t)$ at $t = 0$ is infinite, but with unit integral from $-\infty$ to ∞, i.e. from 0^- to 0^+

  - We also call such functions as singularity function or generalized functions, for more information, please refer to mathematic references

- Relationship

$$\delta(t) = \frac{du(t)}{dt} \qquad \text{—1}^{\text{st}} \text{ derivative}$$

$$u(t) = \int_{-\infty}^{t} \delta(\tau)\, d\tau \qquad \text{—running sum}$$

- Sampling Property

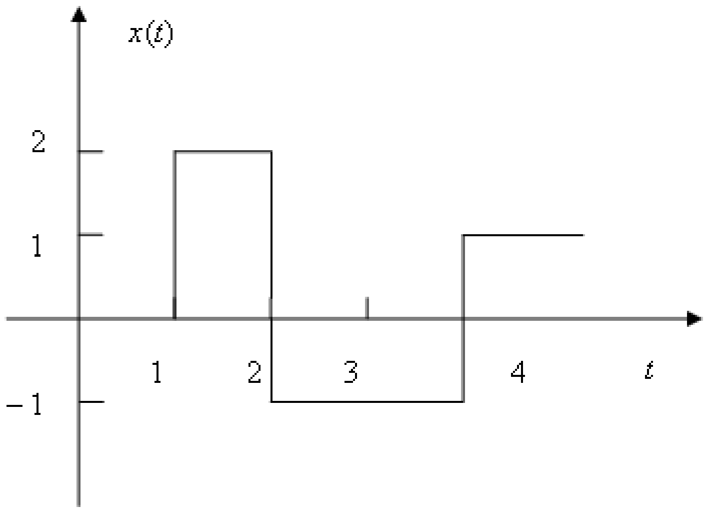
$$x(t)\cdot\delta(t) = x(0)\cdot\delta(t)$$

$$x(t)\cdot\delta(t-t_0) = x(t_0)\cdot\delta(t-t_0)$$

Can we represent x(t) by using a series of unit samples as that for DT signal?

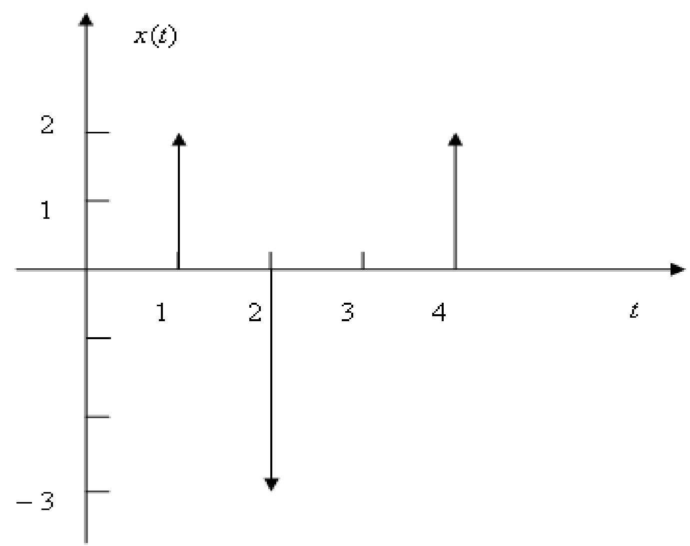
- Scaling Property $\dfrac{d\left(ku(t)\right)}{dt} = k\delta(t)$

- Example: to derive the 1st derivative of x(t)



$$x(t) = 2u(t-1) - 3u(t-2) + 2u(t-4)$$

$$\frac{dx(t)}{dt} = 2\delta(t-1) - 3\delta(t-2) + 2\delta(t-4)$$



- Example: to determine the following signals/values

1. $(t^2 - 1)\delta(t-2)$

2. $\int_{-3}^{3} (t^2 - 1)\delta(t-2)\,dt$

3. $x[n-3]\delta[n+1]$

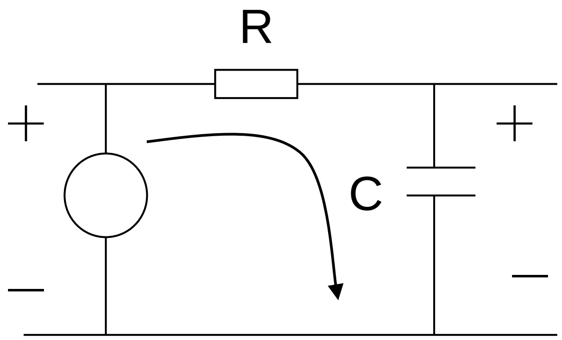
4. $\int_{-3}^{t} (\tau^2 - 1)\delta(\tau-2)\,d\tau$

Topic

- ☐ 1.0 INTRODUCTION
- ☐ 1.1 CONTINUOUS-TIME AND DISCRETE-TIME SIGNALS
- ☐ 1.2 TRASFORMATION OF INDEPENDENT VARIABLE
- ☐ 1.3 EXPONENTIAL AND SINUSOIDAL SIGNALS
- ☐ 1.4 THE UNIT IMPULSE AND UNIT STEP FUNCTIONS
- ☐ 1.5 Definitions and Representations of Systems
- ☐ 1.6 BASIC SYSTEM PROPERTIES

1.5.1 System Modeling

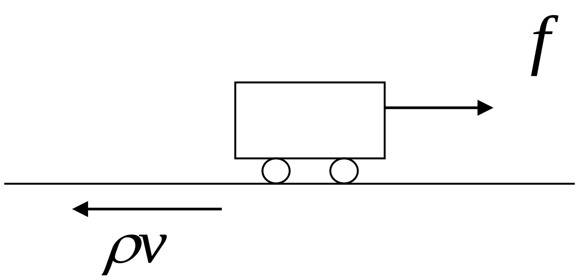
- RLC Circuit



$$\because i(t)=\frac{V_s(t)-V_c(t)}{R} \quad i(t)=C\cdot\frac{dV_c(t)}{dt}$$

$$\therefore \frac{dV_c(t)}{dt}+\frac{1}{RC}V_c(t)=\frac{1}{RC}V_s(t)$$

- Mechanism System



$$\because \frac{dv(t)}{dt}=\frac{1}{m}[f(t)-\rho v(t)]$$

$$\therefore \frac{dv(t)}{dt}+\frac{\rho}{m}v(t)=\frac{f(t)}{m}$$

1.5.1 System Modeling

- Observations:

$$\frac{dy(t)}{dt} + ay(t) = bx(t)$$

  - Very different physical systems may be modeled mathematically in very similar ways.
  - Very different physical systems may have very similar mathematical descriptions.

1.5.1 System Modeling

- Typical Systems and their block illustrations
  - Amplifier

    $y(t)=cx(t)$
  - Adder

    $y(t)=x_1(t)+x_2(t)$
  - Multiplier

    $y(t) =x_1(t)*x_2(t)$
  - Differentiator/Difference

    $y(t)=dx(t)/dt$, $y[n]=x[n]-x[n-1]$
  - Integrator/Accumulator

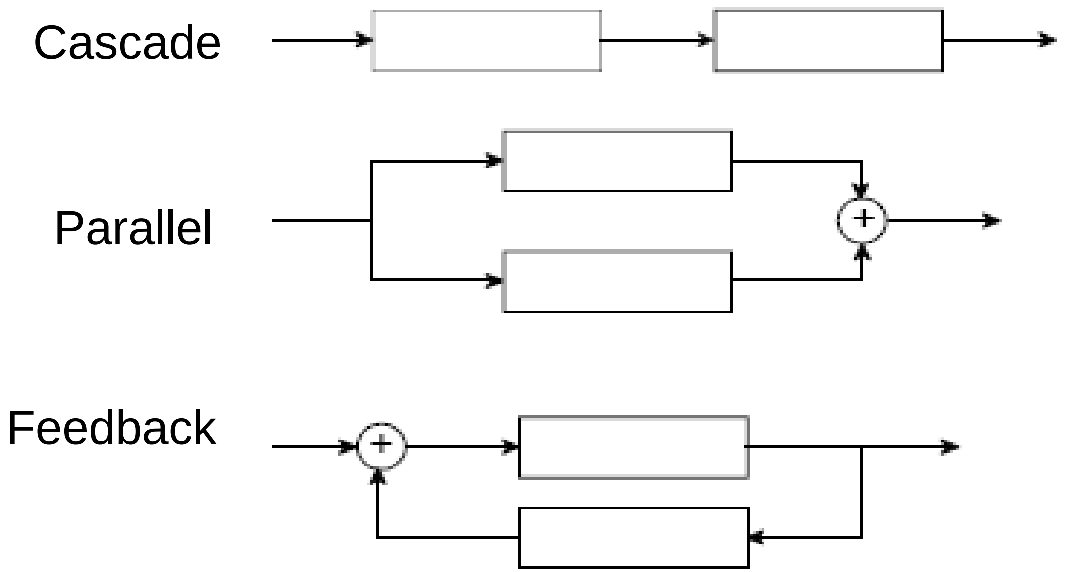
    …

1.5.2 System Analysis

- Memory vs. Memoryless
- Invertibility: Invertible vs. noninvertible
- Causality: Casual vs. non-Casual
- Linearity: Linear vs. non-Linear
- Time-invariance: Time-invariant vs. Time-varying
- Stability: Stable vs. non-Stable

1.5.3 System Interconnections

- The concept of system interconnections
  - To build more complex systems by interconnecting simpler subsystems
  - To modify response of a system
- Signal flow (Block) diagram



Topic

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1.6.1 Systems with and without Memory

- Systems with memory: if the current output of the system is dependent on future and/or past values of the inputs and/or outputs, e.g.:
  - Capacitor system:

$$u(t) = \frac{1}{C}\int_{-\infty}^{t} i(\tau)d\tau \qquad y(t) = \frac{1}{C}\int_{-\infty}^{t} x(\tau)d\tau$$

  - Accumulator system:

$$y[n] = \sum_{k=-\infty}^{n} x[k] \qquad y[n] = \sum_{k=-\infty}^{n-1} x[k] + x[n] = y[n-1] + x[n]$$

- Memoryless systems: if the current output of the system is dependent on the input at the same time, e.g.
  - Identity system:

$$y(t) = x(t) \qquad y[n] = x[n]$$

- Examples: to determine the memory property of the following systems:
  - Amplifier, adder, multiplier
  - Integrator, accumulator, differentiator, time inverse system, time scalar, decimator, interpolator, …

1.6.2 Invertibility: Inverse vs. non-inverse systems

- Inverse systems: distinct inputs lead to distinct outputs, e.g.

$$y(t) = 2x(t) \quad - \quad w(t) = \frac{1}{2}y(t)$$

- Non-inverse systems: distinct inputs may lead to the same outputs, e.g.

$$y(t) = x^2(t) \qquad\qquad y[n] = 0$$

- Importance of the concept: encoding for channel coding or lossless compress

1.6.3 Causality

- A system is causal if the output does not anticipate future values of the input, i.e., if the output at any time depends only on values of the input up to that time
  - All real-time physical systems are causal, because time only moves forward. Effect occurs after cause. (Imagine if you own a non-causal system whose output depends on tomorrow’s stock price.)
  - Causality does not apply to spatially varying signals. (We can move both left and right, up and down.)
  - Causality does not apply to systems processing recorded signals, e.g. taped sports games vs. live show.

1.6.3 Causality

- Mathematical definition: A system $x(t) \to y(t)$ is casual if

when	$x_1(t) \to y_1(t)$	$x_2(t) \to y_2(t)$
and	$x_1(t) = x_2(t)$	for all $t \le t_0$
Then	$y_1(t) = y_2(t)$	for all $t \le t_0$

- If two inputs to a casual system are identical up to some point in time t_0, the corresponding outputs are also equal up to the same time.

- Examples: Considering the causality property of the following signals

$$y(t) = x^2(t-1)$$

$$y(t) = x(t+1)$$

$$y[n] = x[-n]$$

$$y[n] = \left(\frac{1}{2}\right)^{n+1} x^3[n-1]$$

1.6.4 Linearity: Linear vs. non-Linear

- Many systems are nonlinear. For example: many circuit elements (e.g., diodes), dynamics of aircraft, econometric models,…
- But why we investigate linear systems?
  - Linear models represent accurate representations of behavior of many systems (e.g., linear resistors, capacitors, other examples given previously,…)
  - Can often linearize models to examine “small signal” perturbations around “operating points”
  - Linear systems are analytically tractable, providing basis for important tools and considerable insight

- Mathematical definition: A system $x(t) \rightarrow y(t)$ is linear if it has the following additivity property and scaling property （可加性和齐次性）

  If $x_1(t) \rightarrow y_1(t)$ and $x_2(t) \rightarrow y_2(t)$

  Additivity property: $x_1(t) + x_2(t) \rightarrow y_1(t) + y_2(t)$

  Scaling property: $ax_1(t) \rightarrow ay_1(t)$

- Equivalent sufficient and necessary condition: superposition property:

  If $x1(t) \rightarrow y1(t)$ and $x2(t) \rightarrow y2(t)$

  then $ax1(t) + bx2(t) \rightarrow ay1(t) + by2(t)$

- Examples, considering the linearity and causality properties of the following signals：

  $y[n] = x2[n]$ Nonlinear, Causal

  $y(t) = x(2t)$ Linear, Non-causal

1.6.5 Time-invariance (TI):

- Informal definition: a system is time-invariant (TI) if its behavior does not depend on what time it is.

- Mathematical definition:
  - For a DT system: A system $x[n] \rightarrow y[n]$ is TI if for any input $x[n]$ and any time shift n_0,

    If $x[n] \rightarrow y[n]$

    then $x[n-n_0] \rightarrow y[n-n_0]$
  - Similarly for a CT time-invariant system,

    If $x(t) \rightarrow y(t)$

    then $x(t-t_0) \rightarrow y(t-t_0)$

- Examples:

  Considering the time-variance property of the following systems:

  - 1. $y[n]=nx[n]$ Time-varying system
  - 2. $y(t)=x2(t+1)$ Time-invariant system

Consider the periodic property of the output of a Time-invariant system with the input signal of period T

- Suppose $x(t + T) = x(t)$
  and $x(t) \rightarrow y(t)$
  Then by TI: $x(t + T) \rightarrow y(t + T)$.

- 3. $y(t)=\cos(x(t))$ Time-invariant system

- 4. Amplitude modulator:
  $y(t)=x(t)\cos\omega t$ Time-varying system

Linear Time-Invariant (LTI) Systems

- By exploiting the superposition property and time –invariant property, if we know the response of an LTI system to some inputs, we actually know the response to many inputs

$$\text{If} \qquad x_k[n] \rightarrow y_k[n]$$

$$\text{Then} \qquad \sum_k a_k x_k[n] \rightarrow \sum_k a_k y_k[n]$$

- If we can find sets of “basic” signals so that
  - a) We can represent rich classes of signals as linear combinations of these building block signals.
  - b) The response of LTI Systems to these basic signals are both simple and insightful.
- So in this course we will study some powerful analysis tools associated with LTI systems

Stability

- If a system satisfies that the input to the system is bounded, i.e. with finite magnitude, the output is also bounded (BIBO)
- Examples:
  when $|x(t)| < M$, determine whether or not the following systems are stable?

$y(t) = t \cdot x(t)$ Unstable

$y(t) = e^{x(t)}$ Stable

- Homework
  - BASIC PROBLEMS WITH ANSWER: 1.10, 1.11, 1.17, 1.18
  - BASIC PROBLEMS: 1.21, 1.22, 1.25, 1.26, 1.27

Q & A

Many Thanks