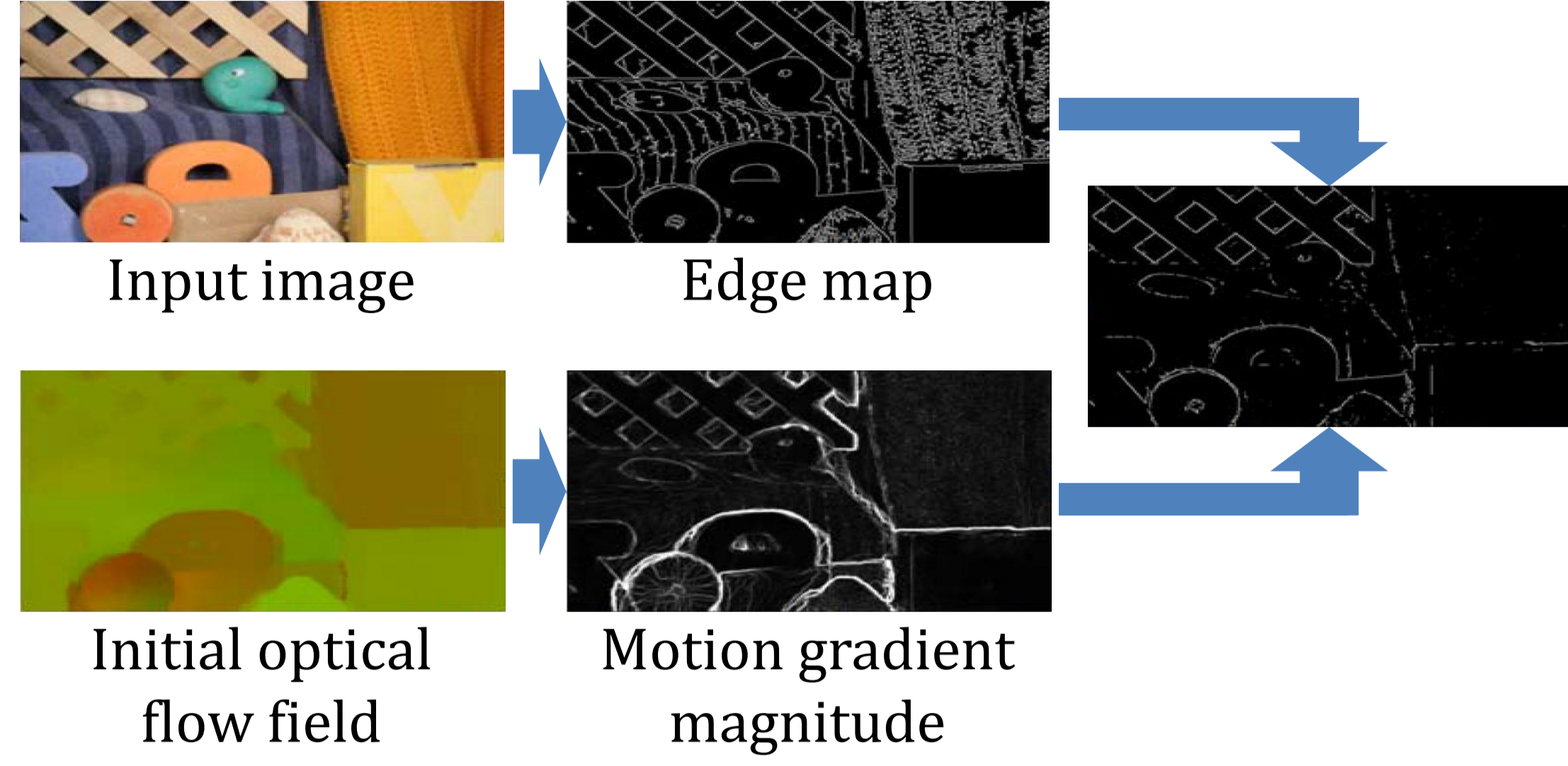


# Optimization of Variational Methods via Motion-based Weight Selection and Keypoint Matching

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## Abstract

One problem in variational optical flow is that it aims at minimizing a global energy function in an iterative manner, but local motion details may be lost. We address this problem by proposing motion-based weight selection and keypoint matching to rectify the global numerical scheme. The selection of the weighting parameter in a self-adaptive and content-aware manner provides a more accurate estimation of the optical flow field near motion boundaries. Motion details and small structures can be preserved in the optical flow field by keypoint matching in the initialization of the optical flow field.



## 1. Variational framework of optical flow

$$\min_{\mathbf{u}} \sum_x \underbrace{|\nabla \mathbf{u}(\mathbf{x})|}_{\text{Total variation regularization term}} + \lambda(\mathbf{x}) \underbrace{|I_1(\mathbf{x}) - I_2(\mathbf{x} + \mathbf{u}(\mathbf{x}))|}_{\text{L1 norm data fidelity term}}$$

$$\min_{\mathbf{u}} \sum_x \left\{ \sum_{d=x,y} \left[ |\nabla u_d| + \frac{(u_d - v_d)^2}{2\theta} \right] + \lambda(\mathbf{x}) |\rho(\mathbf{v})| \right\}$$

where  $\mathbf{v}$  is a close approximate of  $\mathbf{u}$

$$\begin{aligned} \rho(\mathbf{u}) &= I_2(\mathbf{x} + \mathbf{u}) - I_1(\mathbf{x}) \\ &= I_2(\mathbf{x} + \mathbf{u}_0) - I_1(\mathbf{x}) + \langle \nabla I_2(\mathbf{x} + \mathbf{u}_0), \mathbf{u} - \mathbf{u}_0 \rangle \end{aligned}$$

1. For  $\mathbf{v}$  being fixed,

$$\min_{u_d} \sum_x |\nabla u_d| + \frac{1}{2\theta} (u_d - v_d)^2$$

2. For  $\mathbf{u}$  being fixed

$$\min_{\mathbf{v}} \sum_x \lambda(\mathbf{x}) |\rho(\mathbf{v})| + \frac{(u_x - v_x)^2 + (u_y - v_y)^2}{2\theta}$$

The solution of the original objective function can be obtained by iteratively solving the two sub problems.

### Solution to problem 1

$$\mathbf{u}_d = \mathbf{v}_d + \text{div } \mathbf{p}_d$$

where **div** is the discrete divergence, and  $\mathbf{p}$  can be obtained by

$$\mathbf{p}' = \mathbf{p}_d^{(k)} + \frac{\tau}{\theta} \nabla \mathbf{u}_d$$

$$\mathbf{p}_d^{(k+1)} = \frac{\mathbf{p}'}{\max\{|\mathbf{p}'|, 1\}}$$

where  $\tau \leq 1/4$

### Solution to problem 2

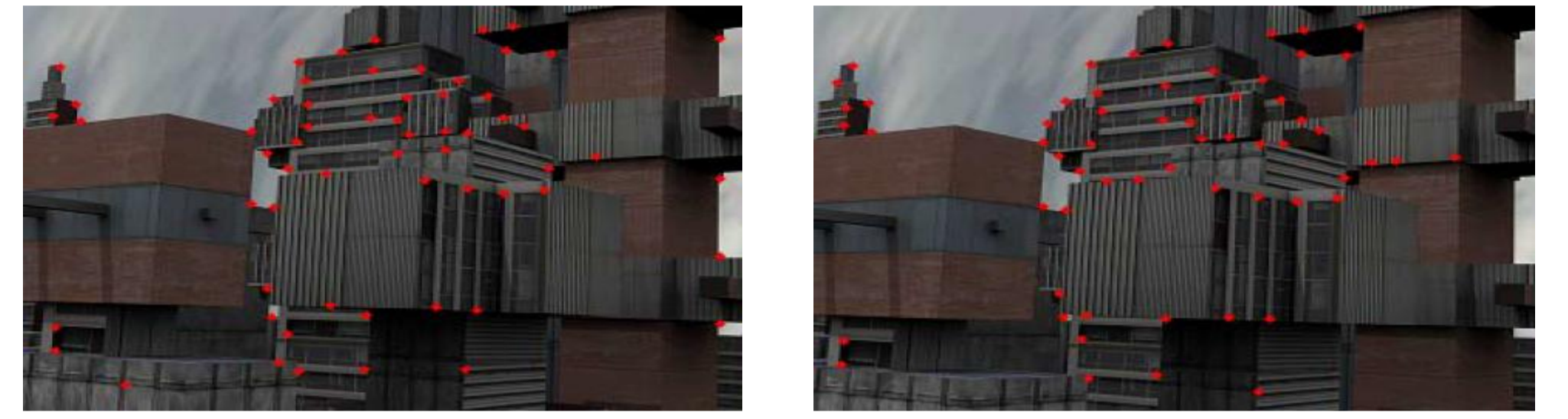
$$\mathbf{v} = \mathbf{u} + \begin{cases} \lambda(\mathbf{x})\theta\nabla I_2, & \text{if } \rho(\mathbf{u}) < -\lambda(\mathbf{x})\theta|\nabla I_2|^2 \\ -\lambda(\mathbf{x})\theta\nabla I_2, & \text{if } \rho(\mathbf{u}) > \lambda(\mathbf{x})\theta|\nabla I_2|^2 \\ -\frac{\rho(\mathbf{u})\nabla I_2}{|\nabla I_2|^2}, & \text{if } |\rho(\mathbf{u})| \leq \lambda(\mathbf{x})\theta|\nabla I_2|^2 \end{cases}$$

## 2. Motion-based weight selection

Motion boundaries have larger motion variations, therefore we allow relatively larger regularization term (i.e., larger  $\lambda$ ) near motion boundaries.

## 3. Rectification via keypoint matching

### (a) Keypoint extraction



$$KP_1 = \{p_1^1, \dots, p_n^1\}$$

$$KP_2 = \{p_1^2, \dots, p_m^2\}$$

### (b) Keypoint matching

The best matched keypoint for  $p_i^1$  in  $KP_2$  is

$$j = \underset{j}{\operatorname{argmin}} |F_i^1 - F_j^2|$$

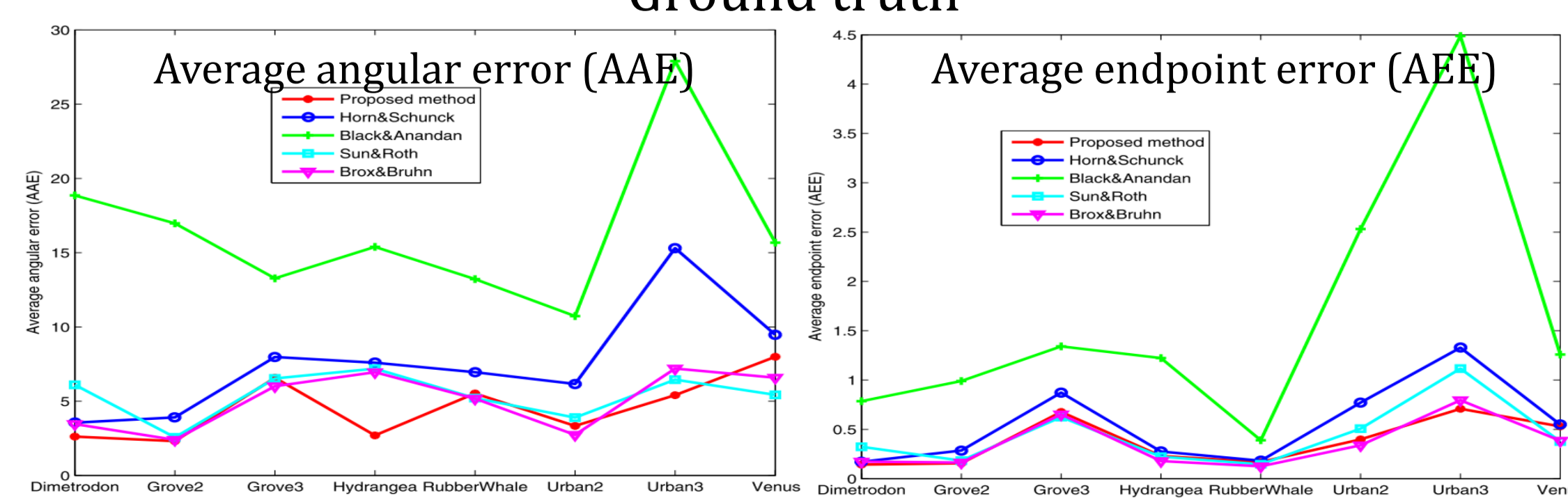
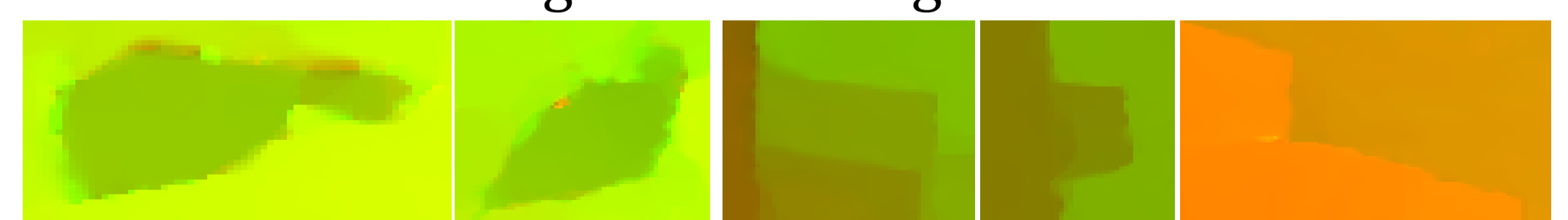
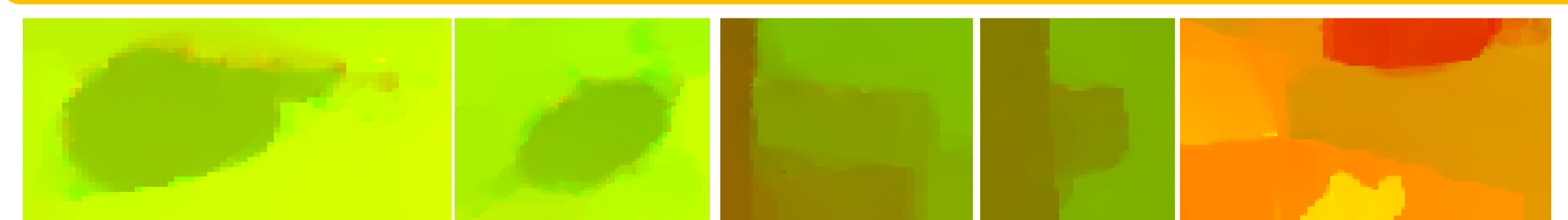
$$\text{s.t. } \begin{cases} |x_i^1 - x_j^2| \leq R & R: \text{radius of neighborhood} \\ |F_i^1 - F_j^2| < C & C: \text{maximum matching cost} \end{cases}$$



### (c) Optical flow vectors by keypoint matching

$$\mathbf{u}(x_i^1) = x_j^2 - x_i^1$$

## 4. Experimental result



Performance comparison with state-of-the-art methods